Symblicit Abstraction of Continuous-Time Markov Models for Transient Rewards

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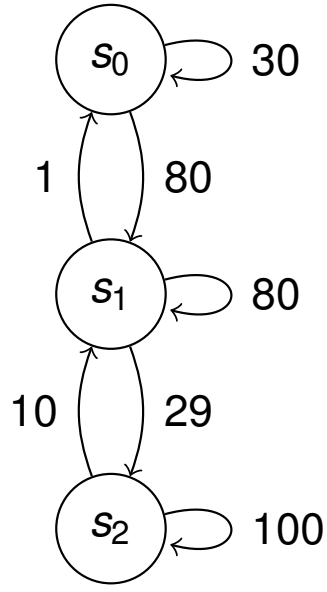
ROCKS, April 27th, 2012

Problem



$$\begin{pmatrix} 30 & 80 & 0 \\ 1 & 80 & 29 \\ 0 & 10 & 100 \end{pmatrix}$$

- continuous-time Markov chains occur in many areas
  - performance evaluation
  - biology
  - ...

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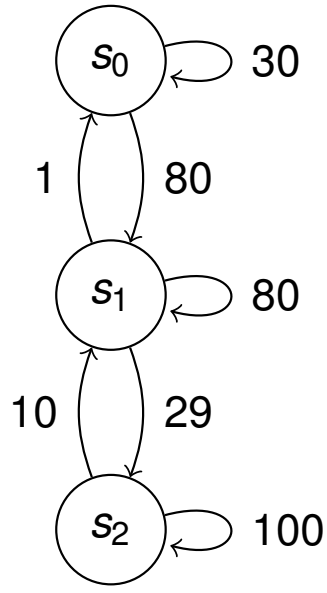

$$\begin{pmatrix} 30/110 & 80/110 & 0 \\ 1/110 & 80/110 & 29/110 \\ 0 & 10/110 & 100/110 \end{pmatrix} \cdot \begin{pmatrix} 0.4 \\ 0.7 \\ 0.2 \end{pmatrix} = \begin{pmatrix} next_0 \\ next_1 \\ next_2 \end{pmatrix}$$

- continuous-time Markov chains occur in many areas
  - performance evaluation
  - biology
  - ...
- model well understood
- efficient analysis techniques exist
  - often based on repeated matrix-vector multiplications

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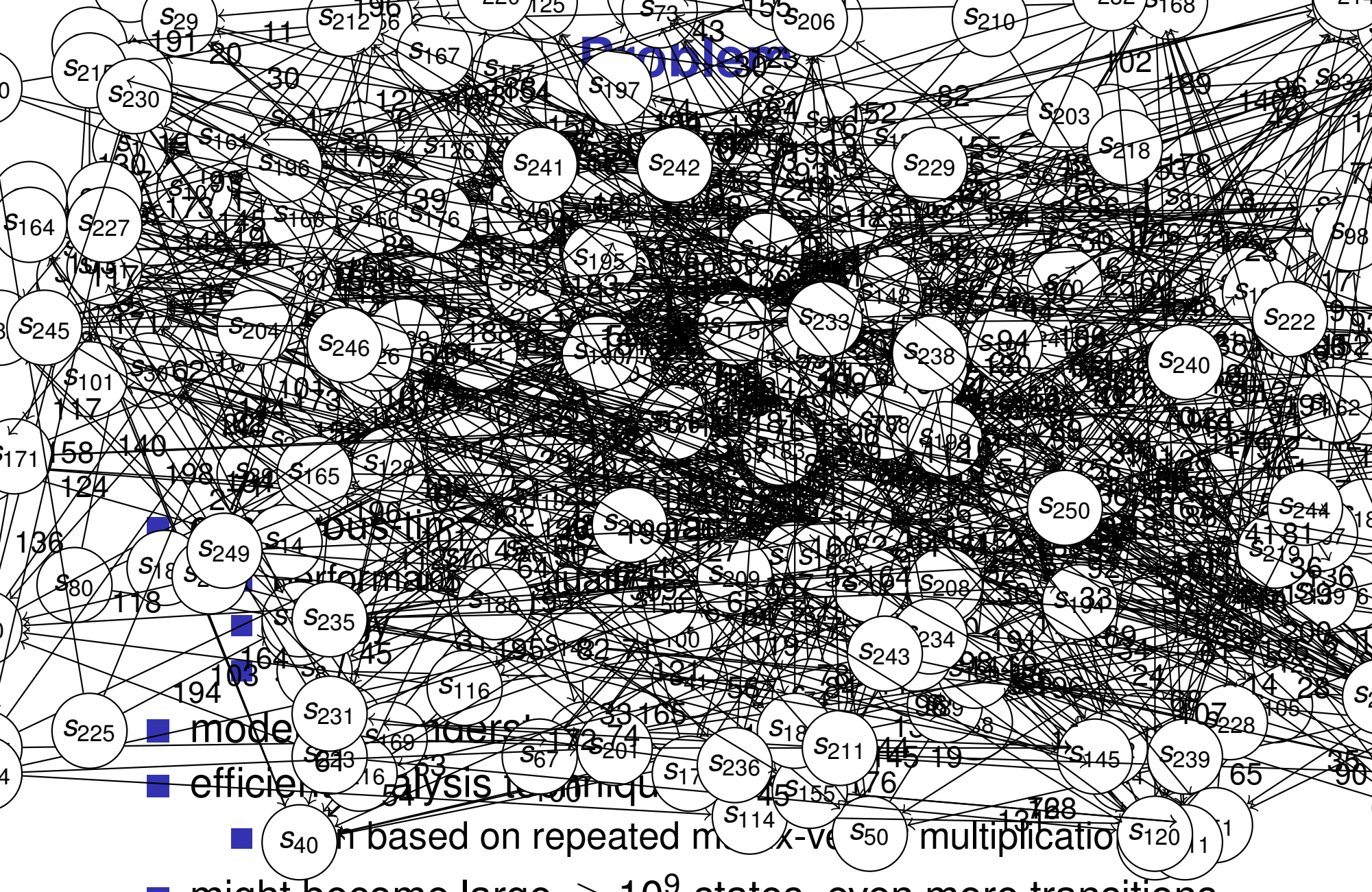
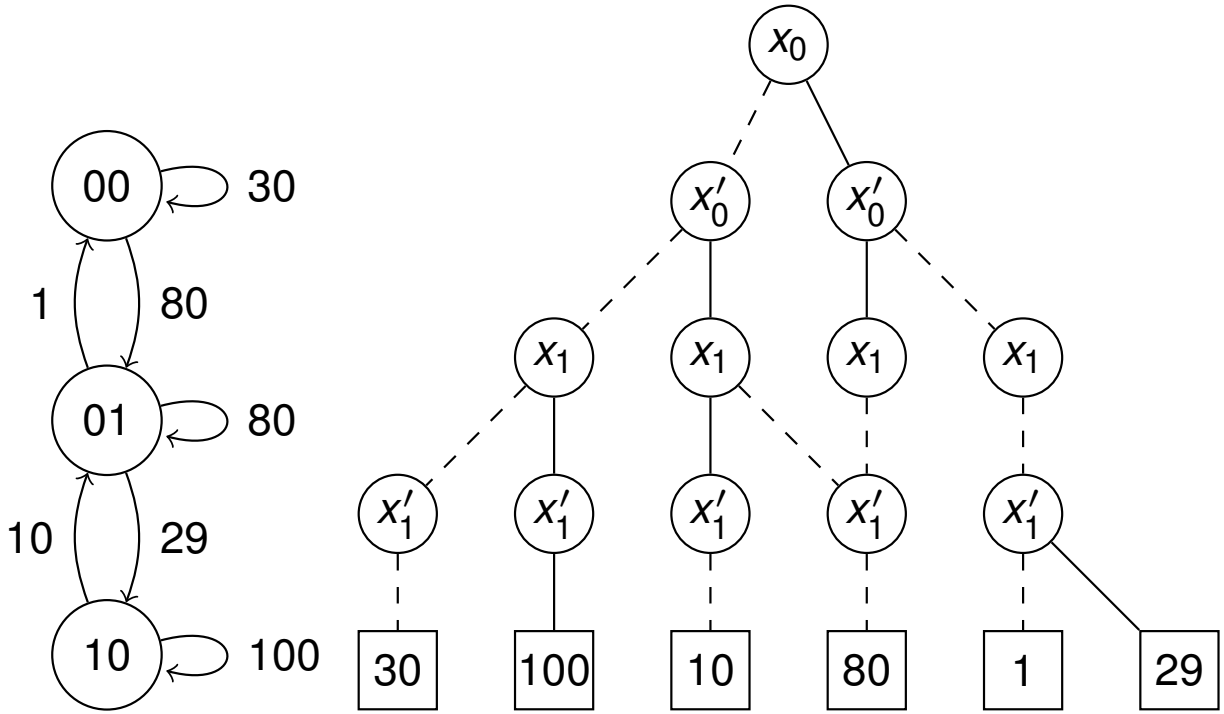


- model checkers' performance
- efficient analysis is important
  - based on repeated matrix-vector multiplication
- might become large, $\geq 10^9$ states, even more transitions
- $\Rightarrow$ direct analysis often not possible

Existing methods (1/2)

Multi-terminal binary decision diagrams (MTBDDs)



- extension of BDDs with more than two terminal nodes

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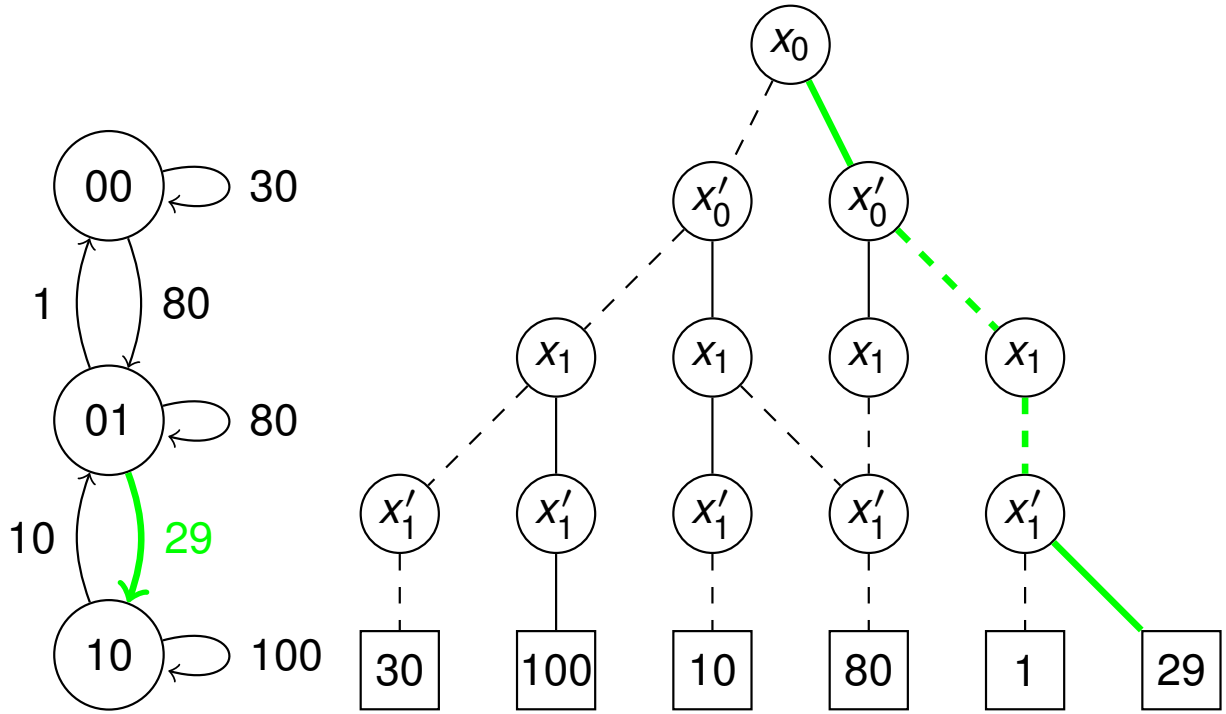

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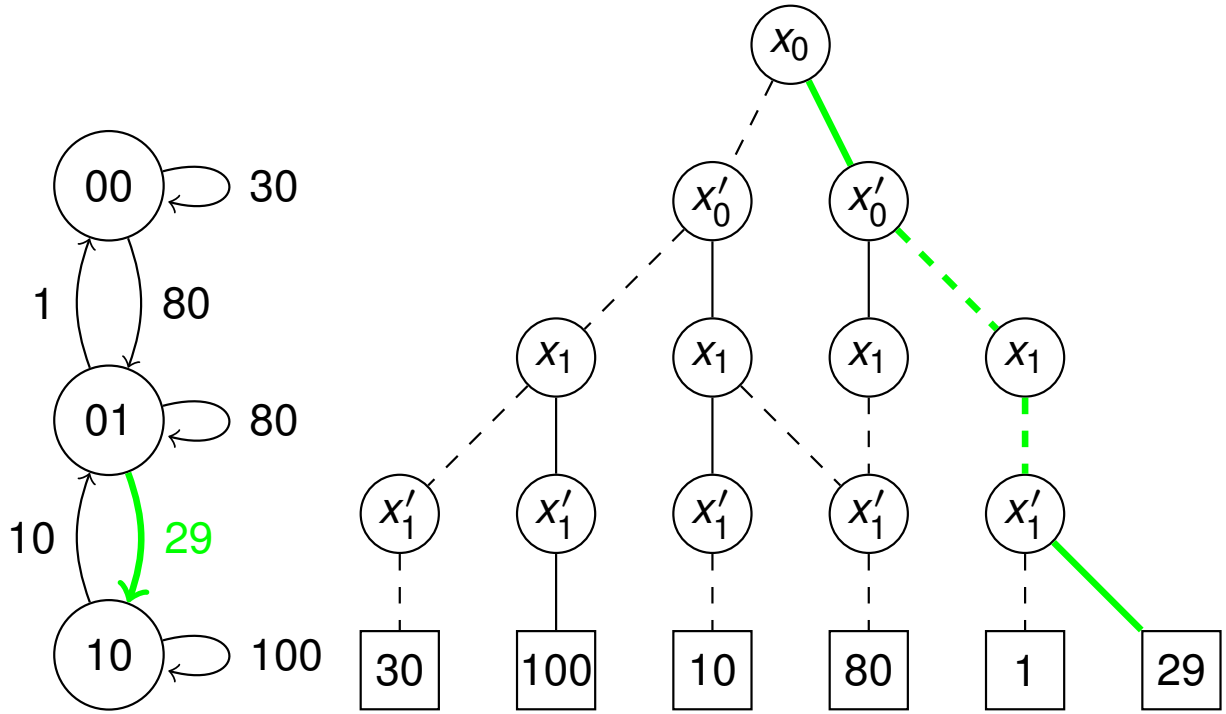

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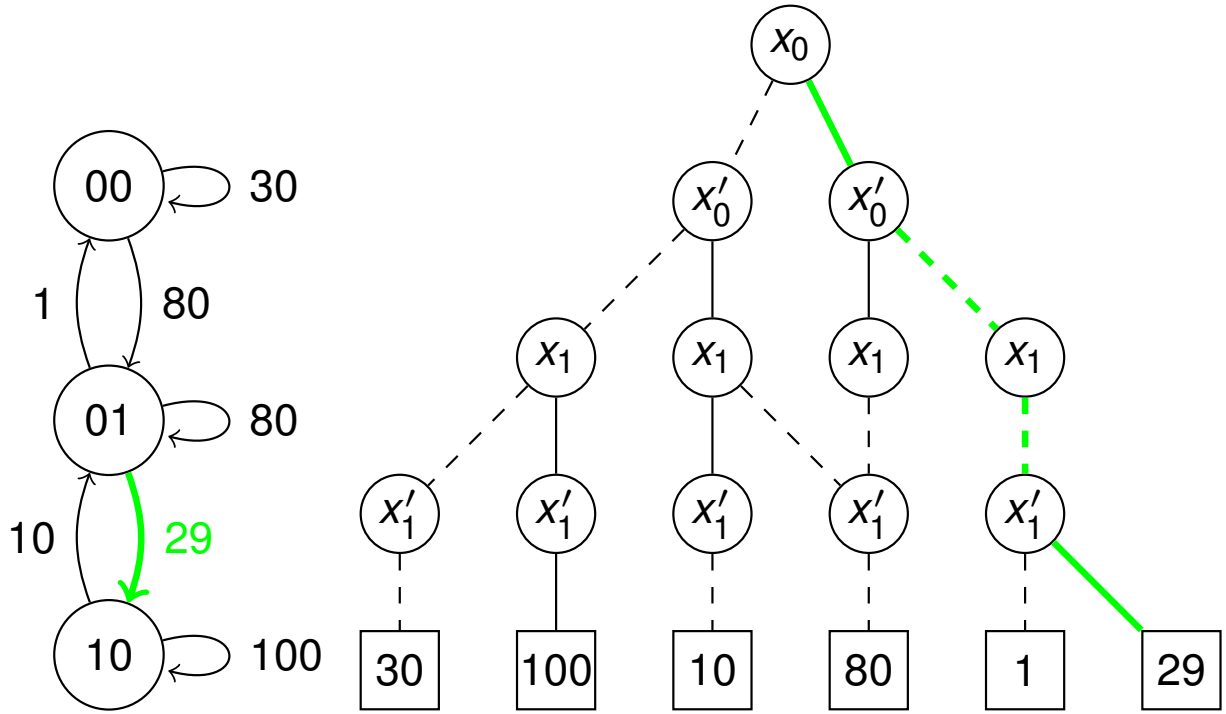

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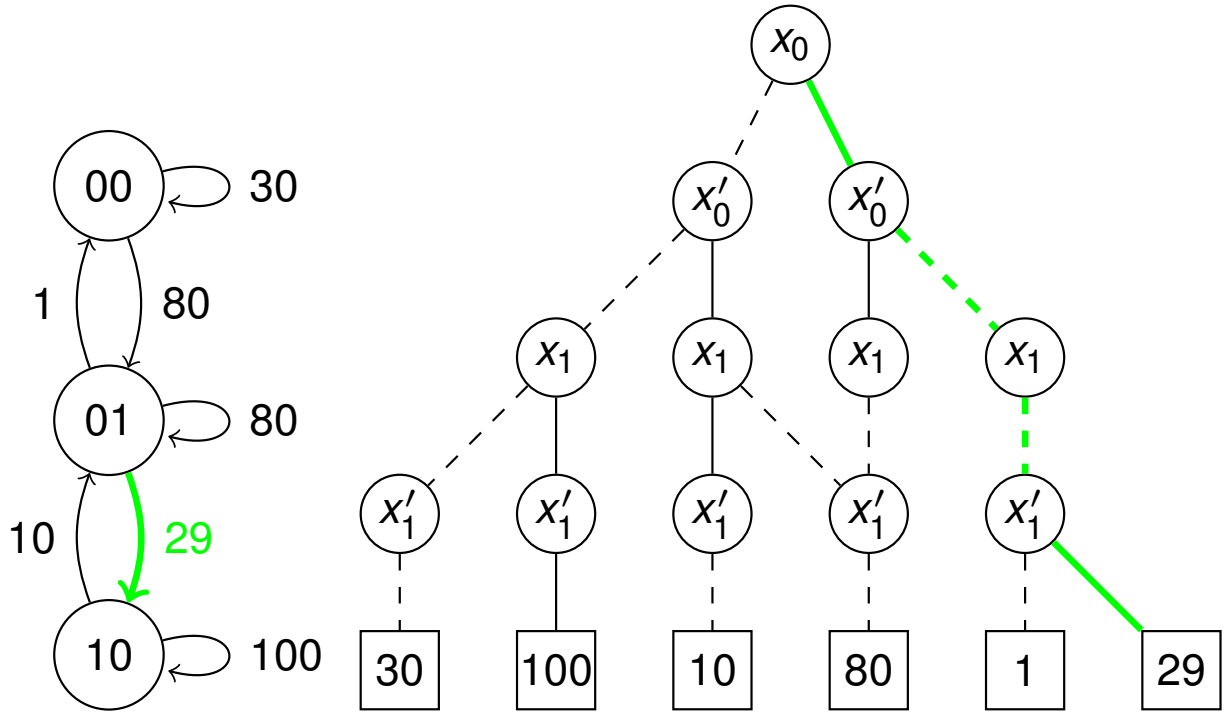

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- to store transition matrix and for matrix-vector mult.
- works very well in some cases
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- does not work well if many different rates/probabilities
  - happens often in value iteration

Existing methods (2/2)

Hybrid method

$$\begin{pmatrix} 0.4 \\ 0.7 \\ 0.2 \end{pmatrix}$$
$$\begin{pmatrix} next_0 \\ next_1 \\ next_2 \end{pmatrix}$$

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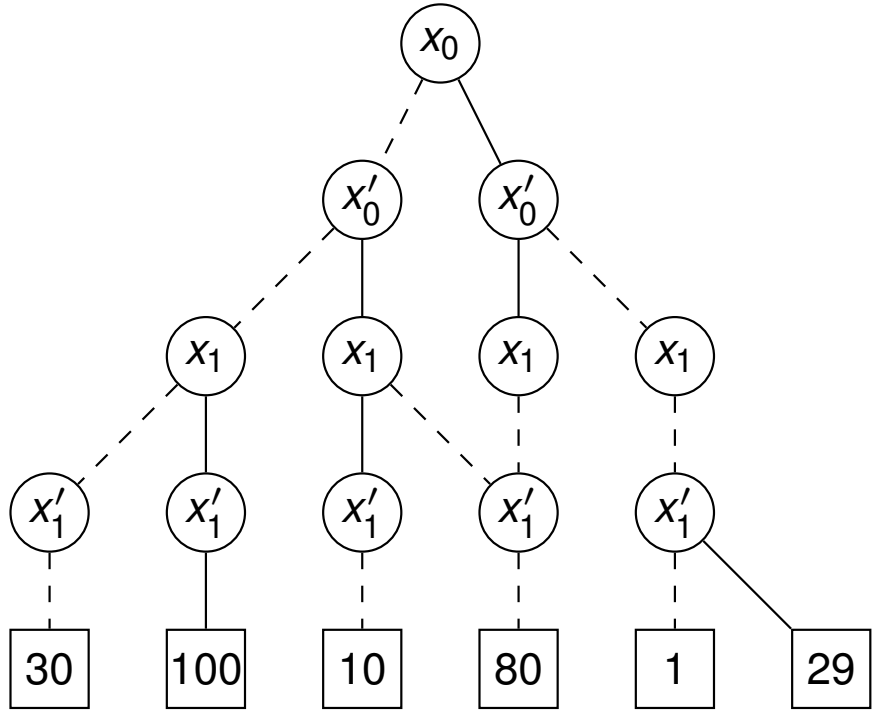

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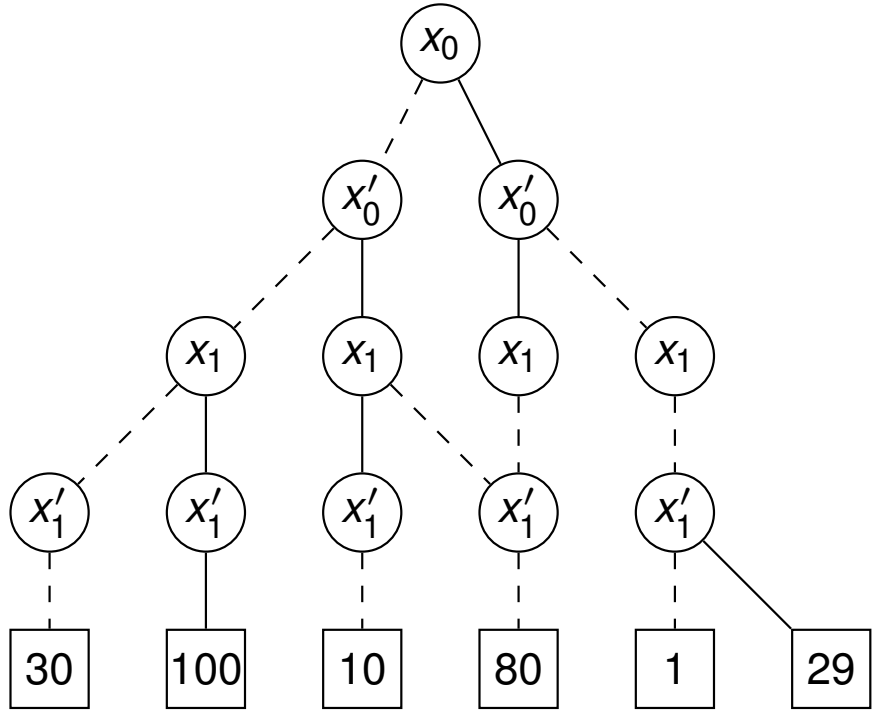

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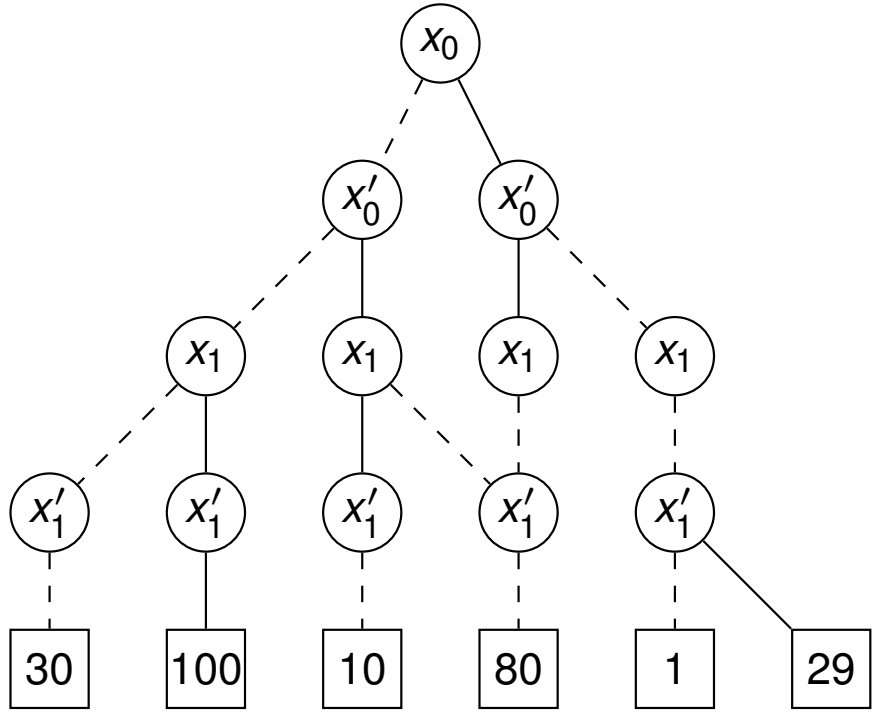

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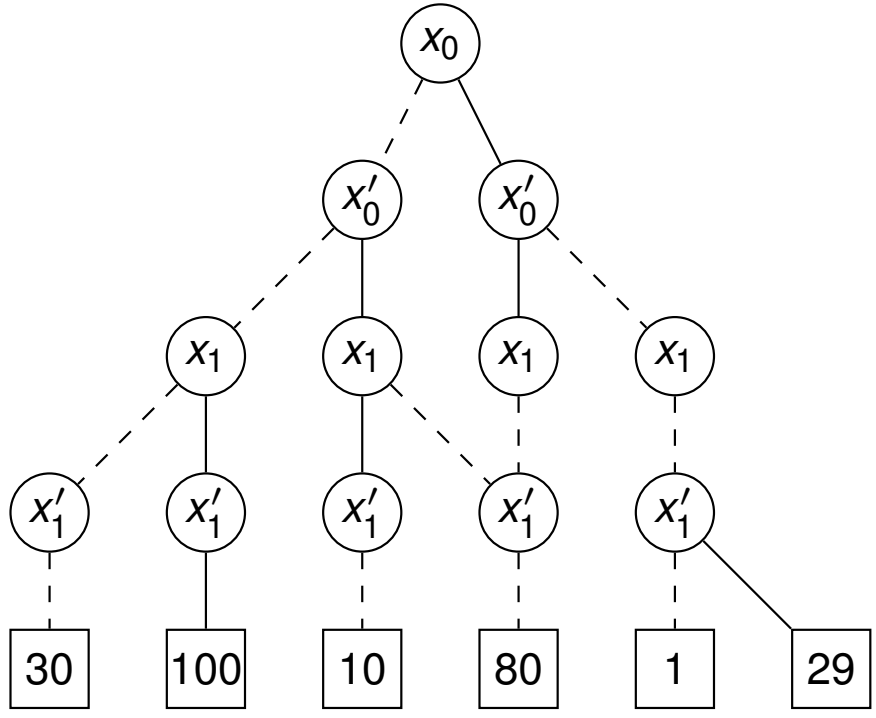

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- and transition matrix in BDD-like way
- can reduce memory usage for transitions
- might be slow
- not applicable in case of very large state sets

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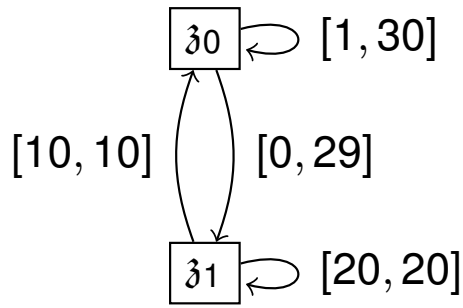
- $\Rightarrow$ **abstract continuous-time Markov chains (ACTMCs)** (by Klink et al.) to abstract CTMCs, special case of continuous-time Markov decision processes (CTMDPs)



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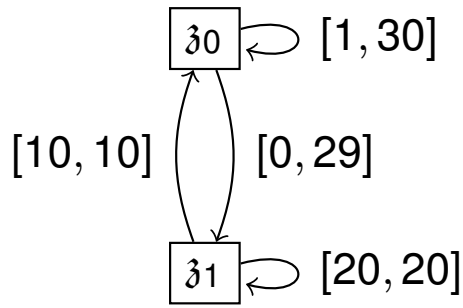

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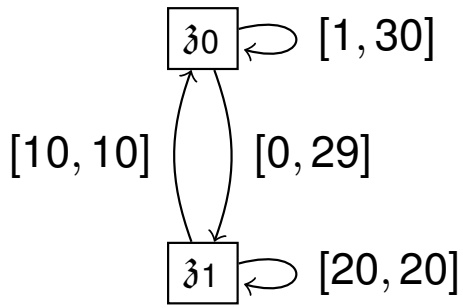

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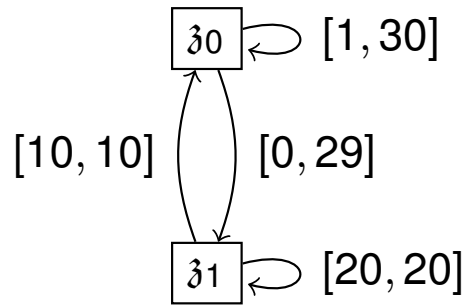

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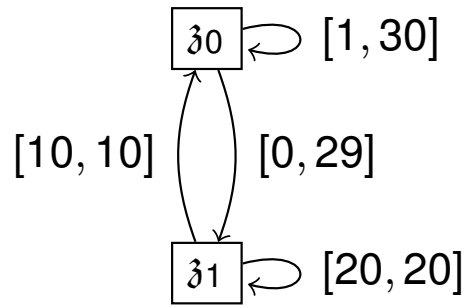

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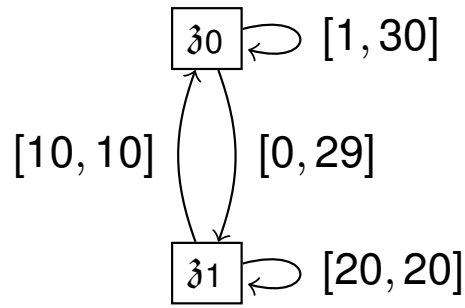

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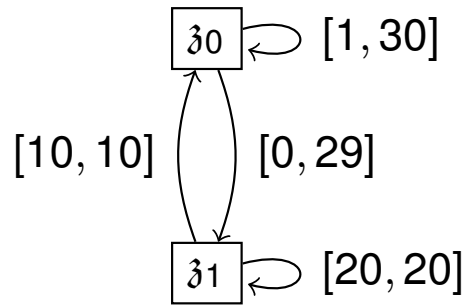

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    - which states to subsume?

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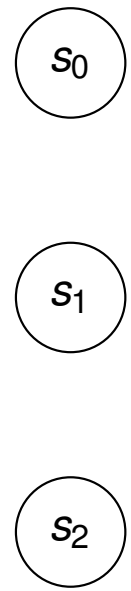

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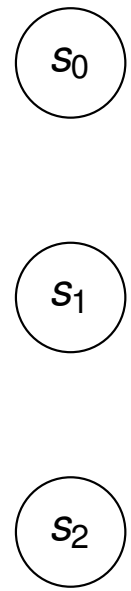

$\mathbf{u} = 110$

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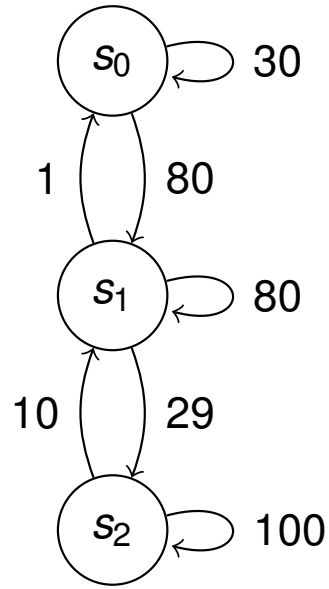

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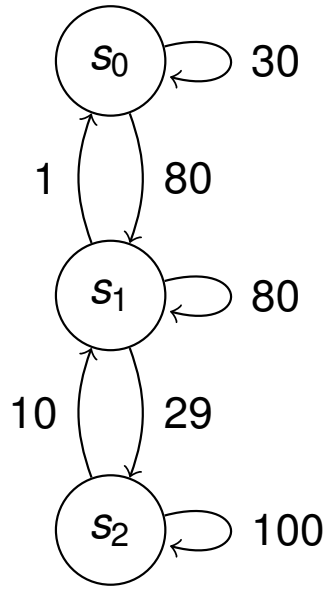

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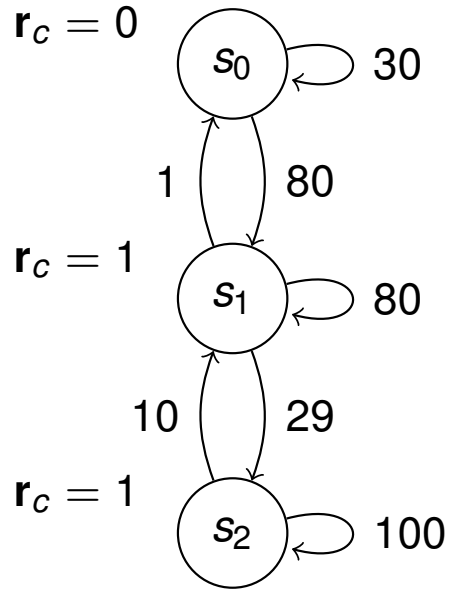

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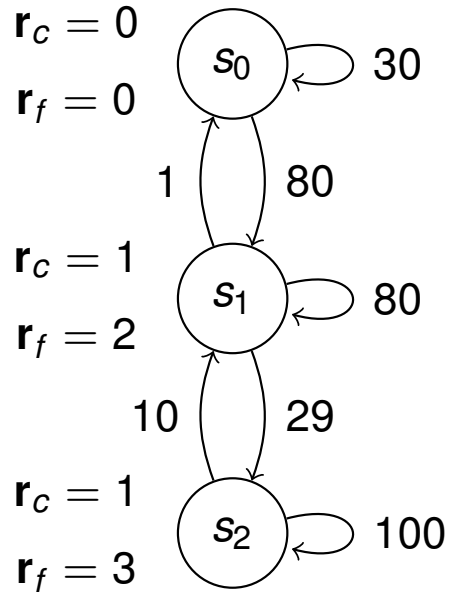

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(A) Properties: Behaviour and Value

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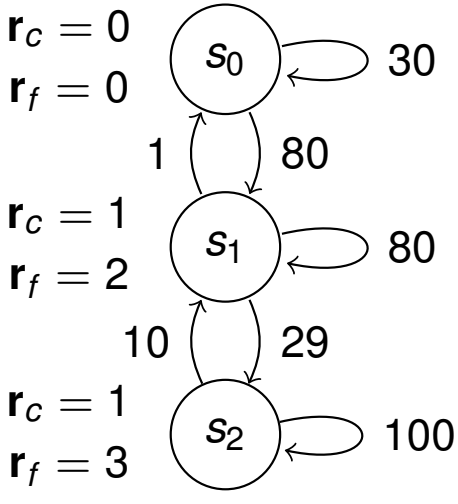

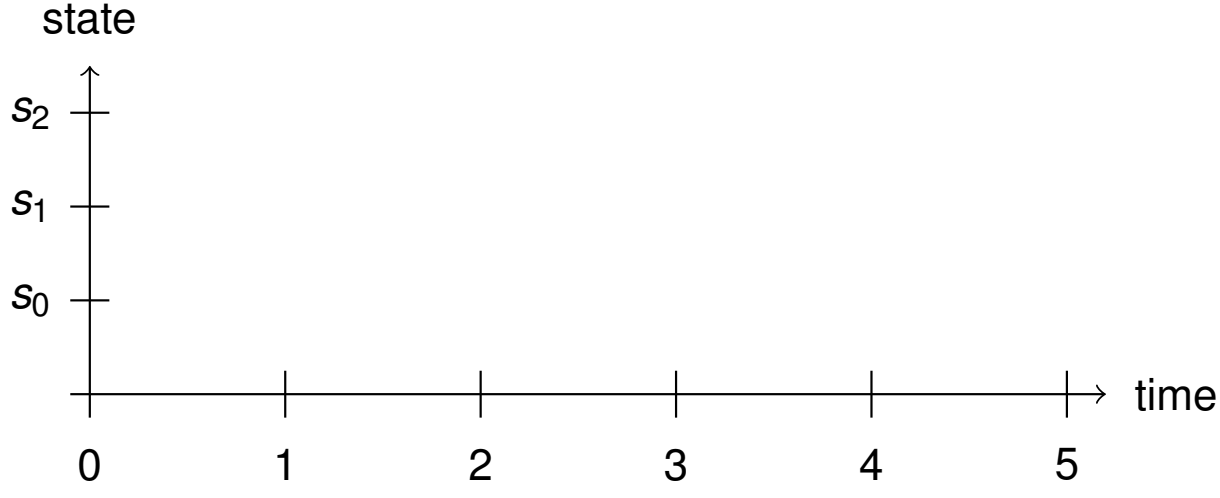

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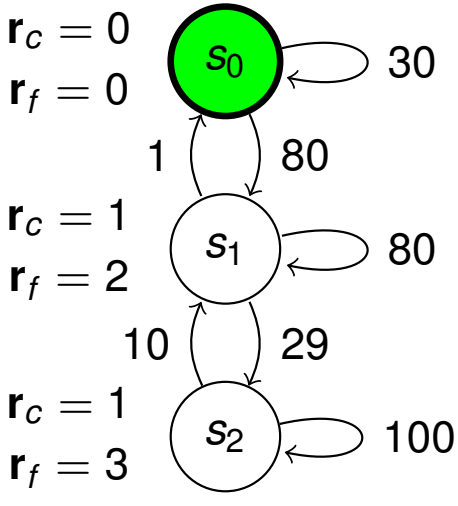

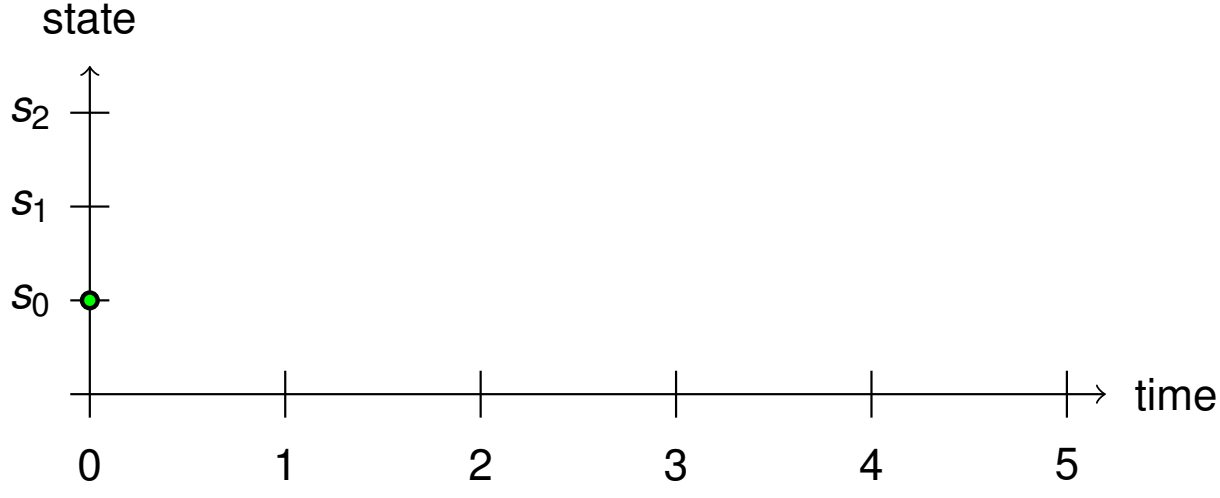

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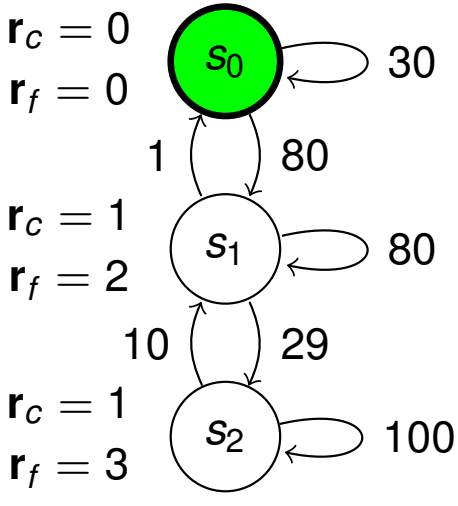

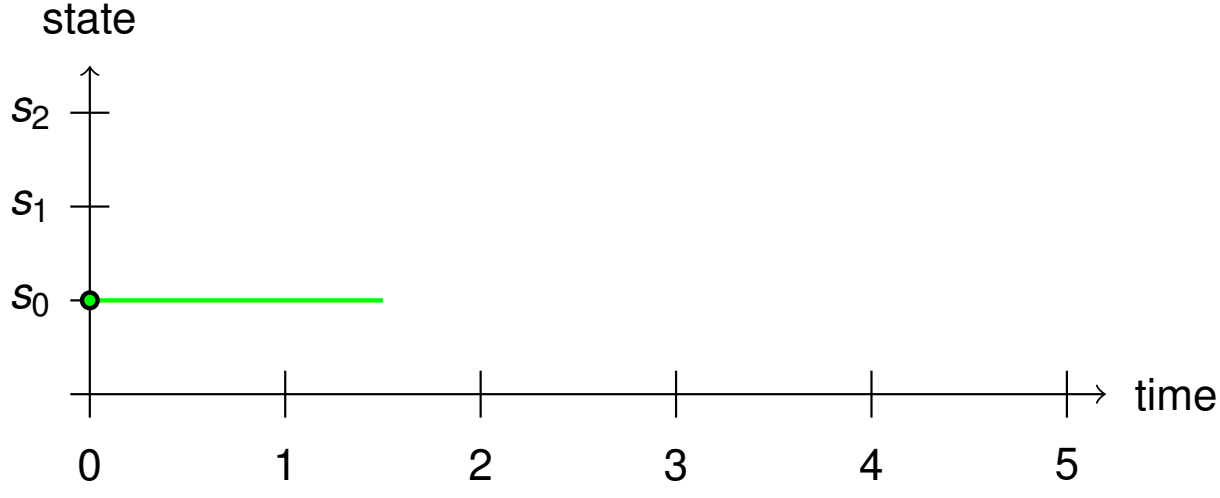

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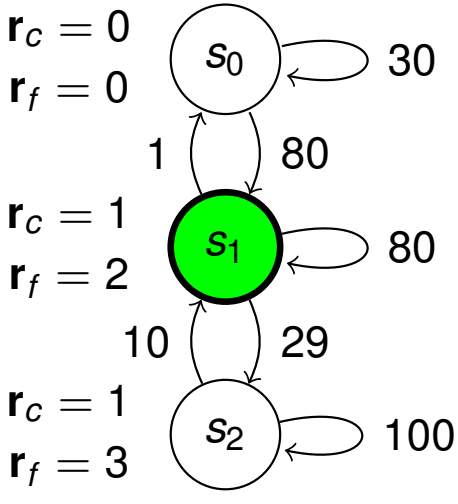

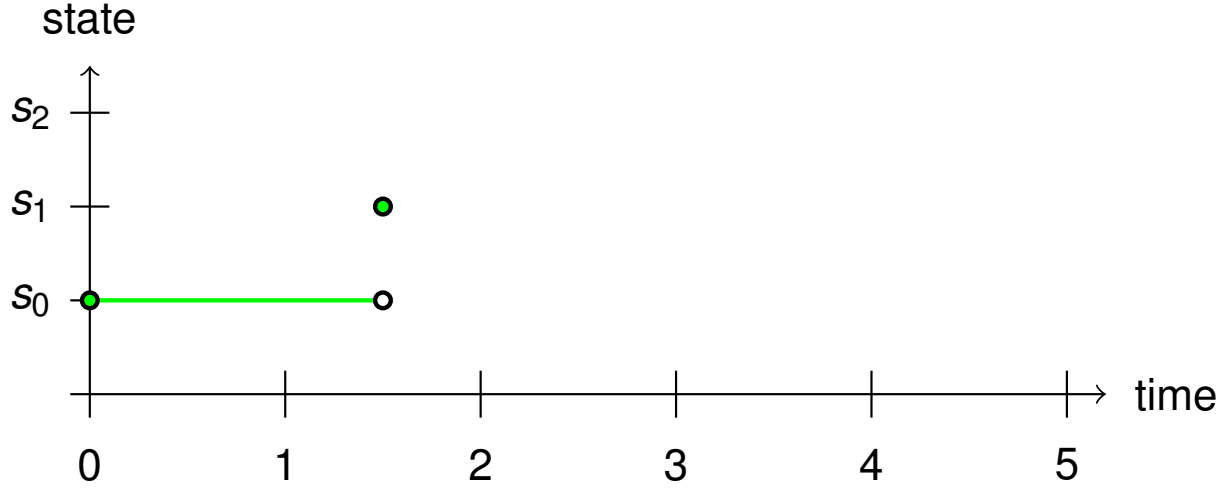

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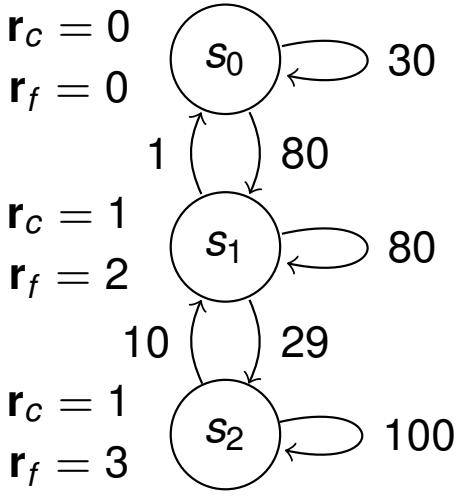
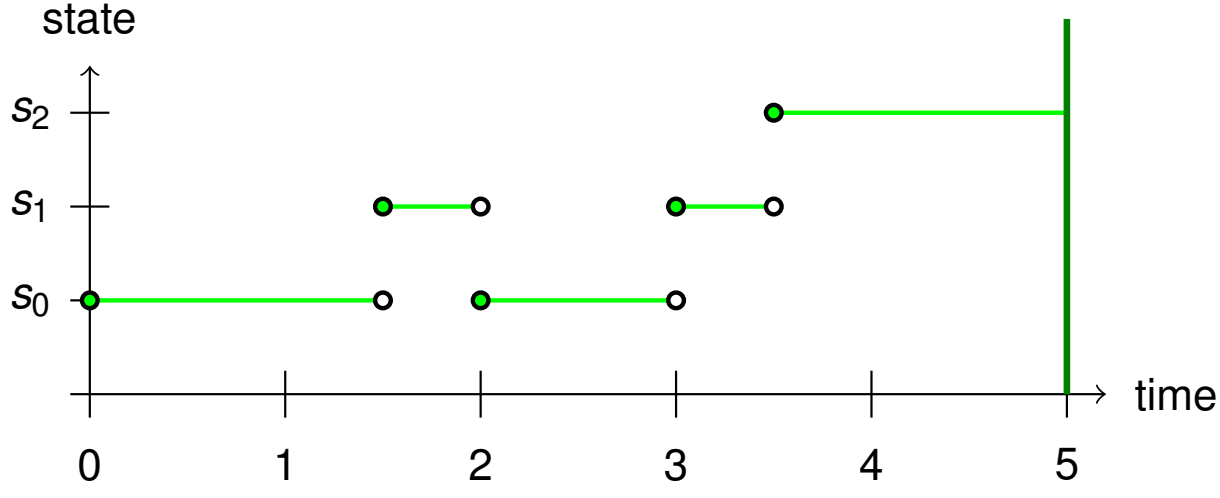
  - until some given $\mathbf{t} > 0$





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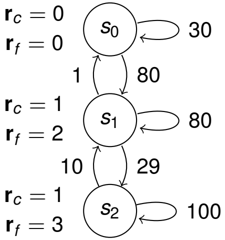
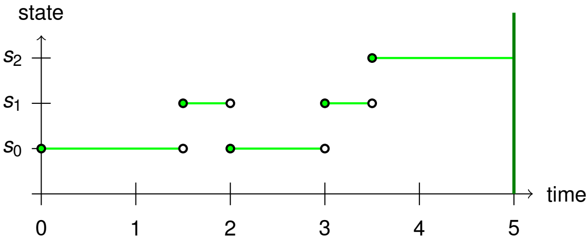
$$\underbrace{\mathbf{E}}_{\text{over paths}} \left[\qquad\qquad\qquad\qquad \right]$$





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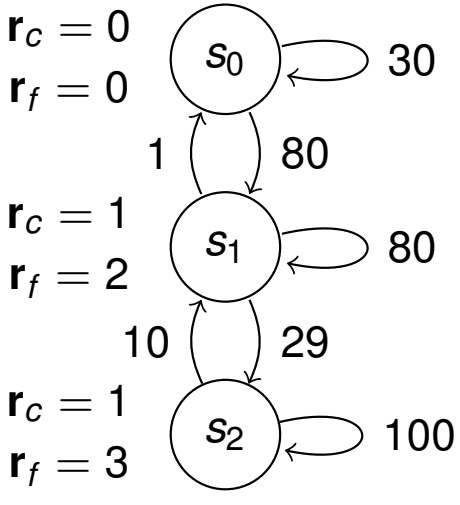
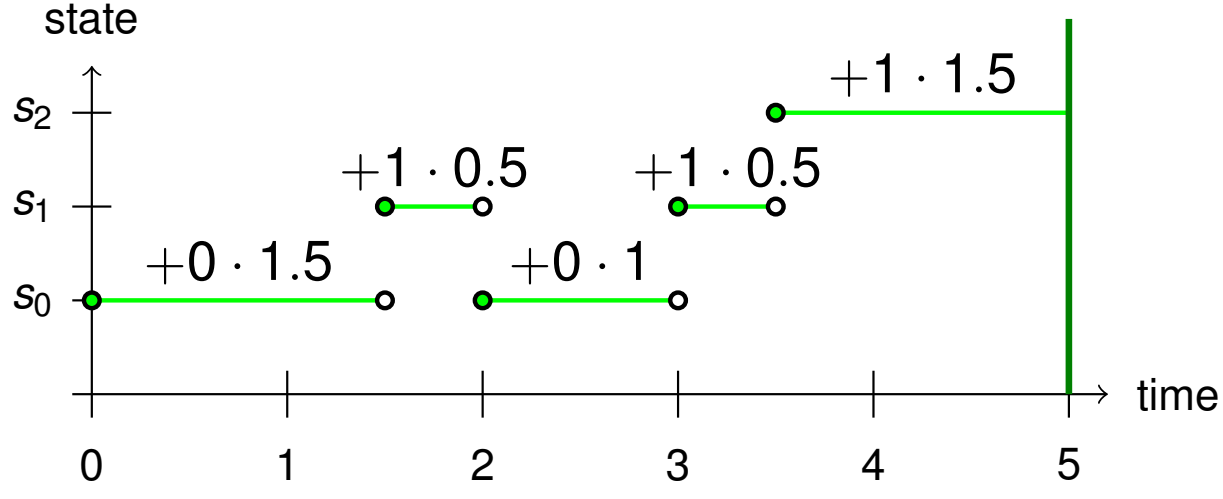
$$\underbrace{\mathbf{E}}_{\text{over paths}} \left[\underbrace{\int_0^{\mathbf{t}} \mathbf{r}_c(X_u)\,\mathrm{d}u}_{\text{accumulated until } \mathbf{t}} \right]$$

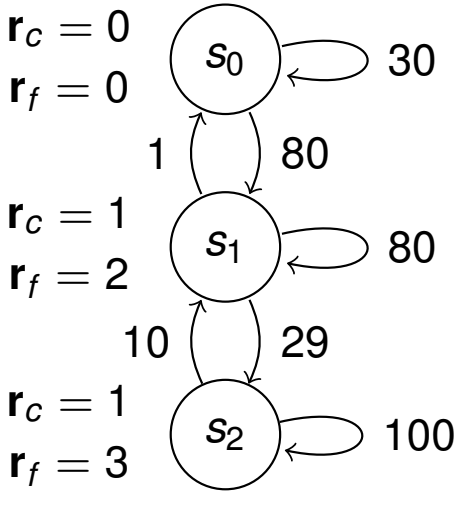


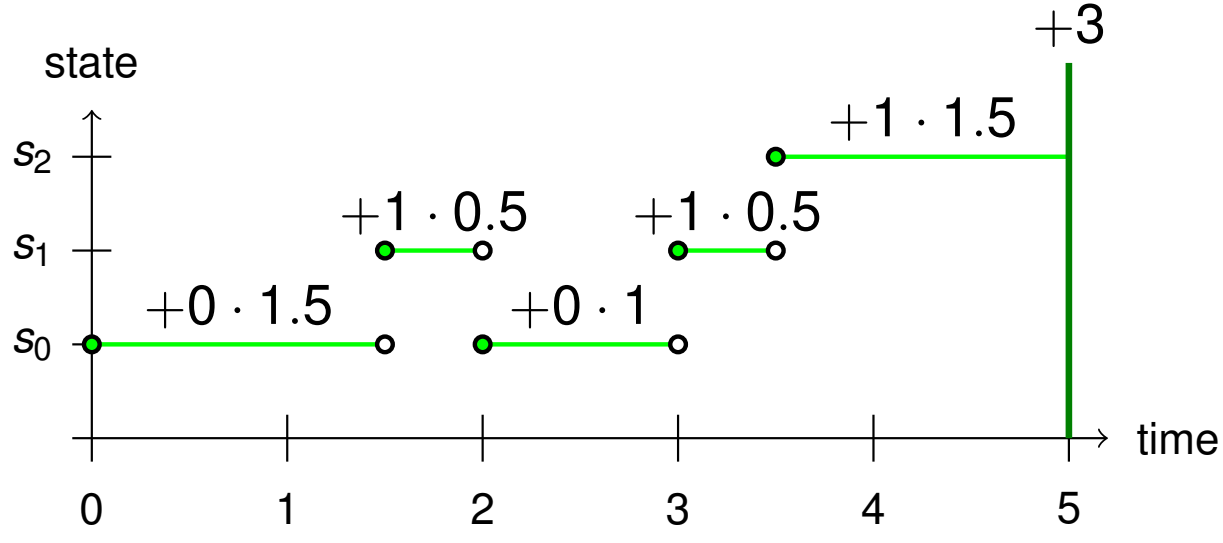


(A) Properties: Behaviour and Value

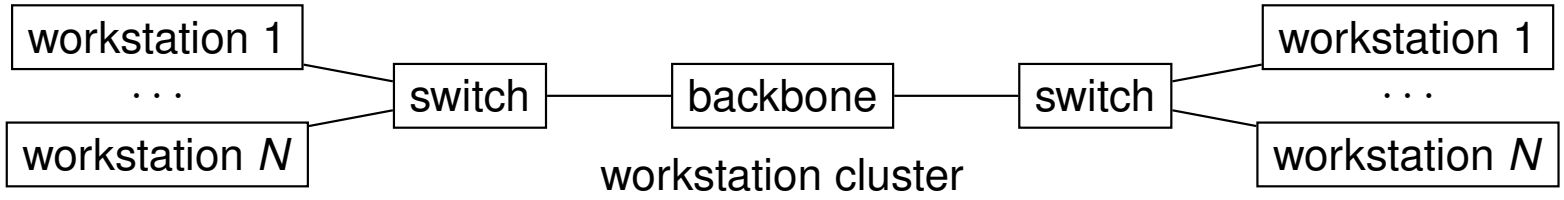
- model behaviour X_t
  - start in given s_0
  - wait for t_0 chosen by exponential distribution with param. **u**
  - move to s_1 with prob. $\mathbf{R}(s_0, s_1)/\mathbf{u}$
  - until some given $\mathbf{t} > 0$
- problem: compute **value**

$$\underbrace{\mathbf{E}}_{\text{over paths}} \left[\underbrace{\int_0^{\mathbf{t}} \mathbf{r}_c(X_u)\,\mathrm{d}u}_{\text{accumulated until } \mathbf{t}} + \underbrace{\mathbf{r}_f(X_{\mathbf{t}})}_{\text{obtained at } \mathbf{t}} \right]$$





(A) Properties: Why is this useful?



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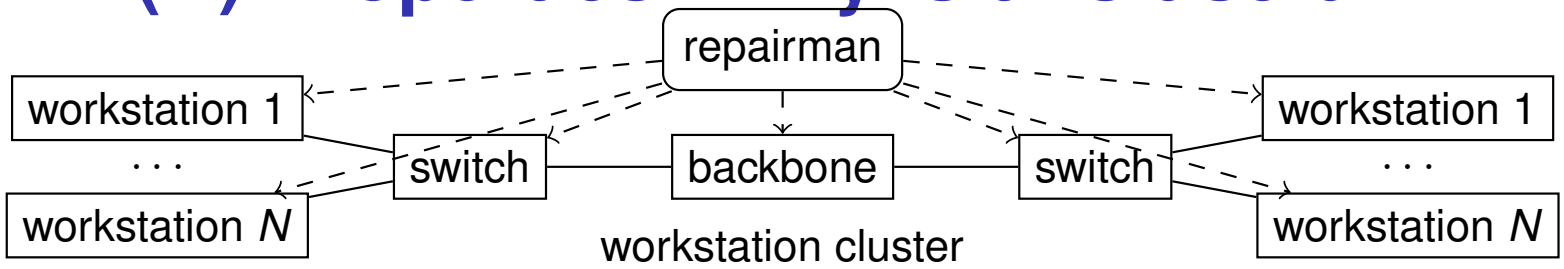

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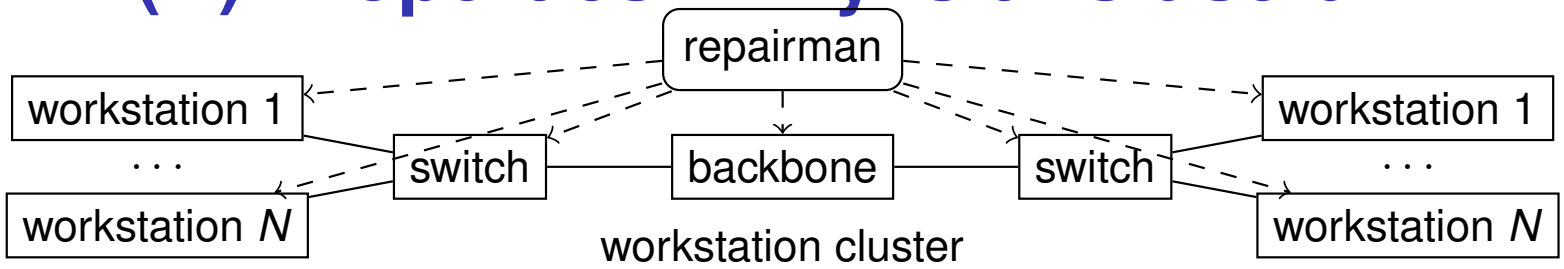

- cumulative reward
  - #repairs until **t**

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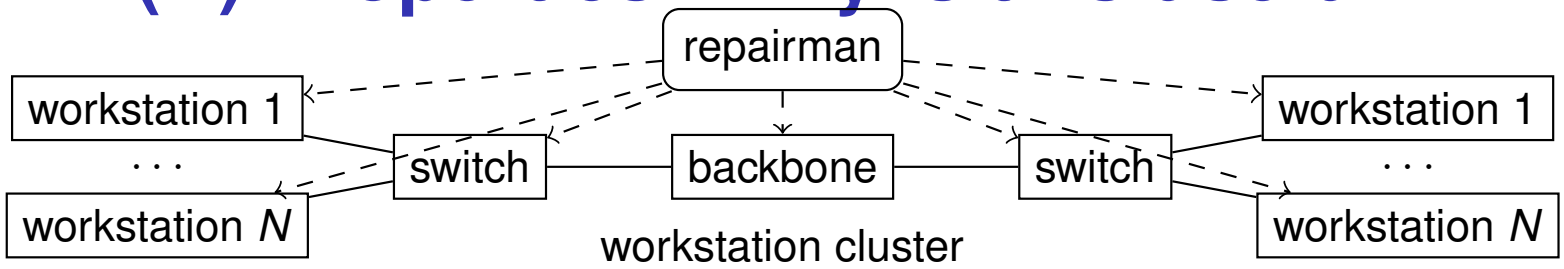

- cumulative reward
  - #repairs until **t**
- final reward
  - #workstations up and running at **t**

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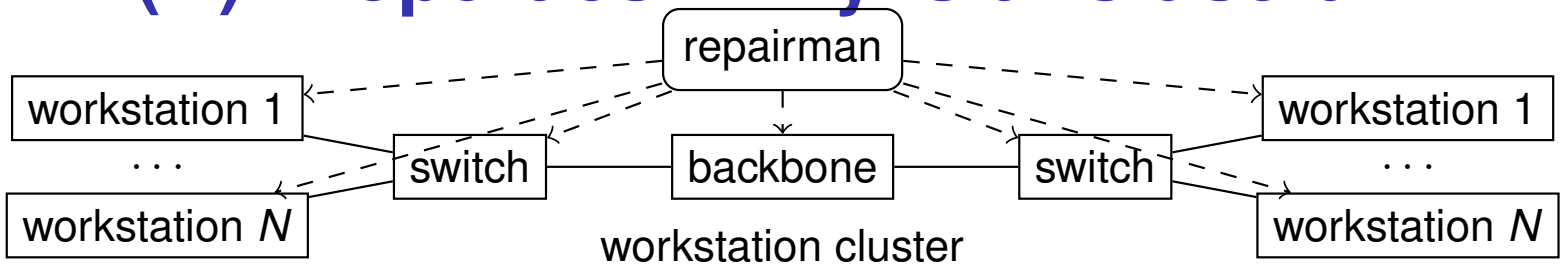

- cumulative reward
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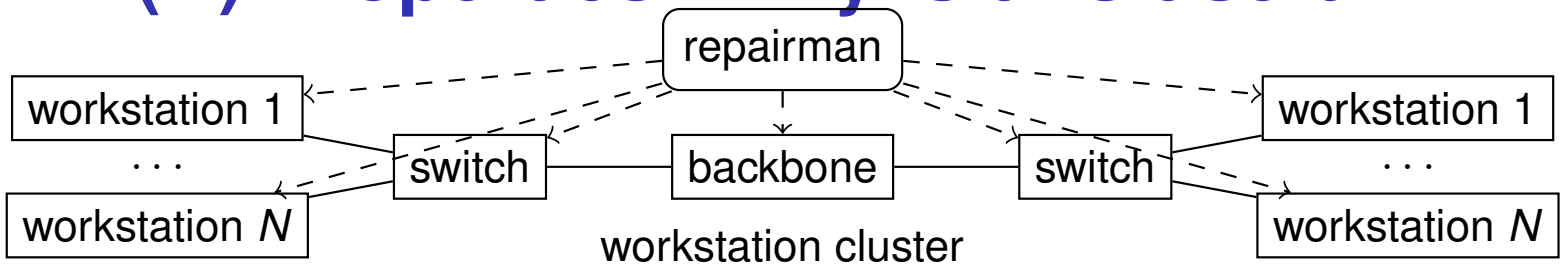

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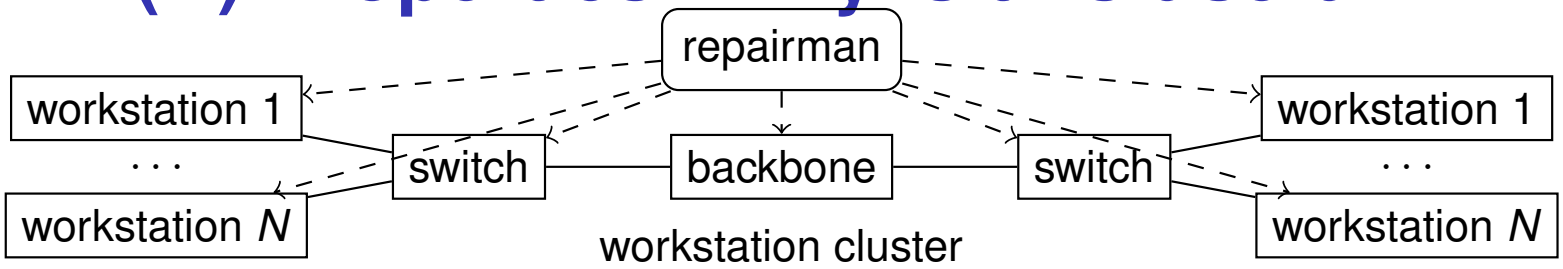

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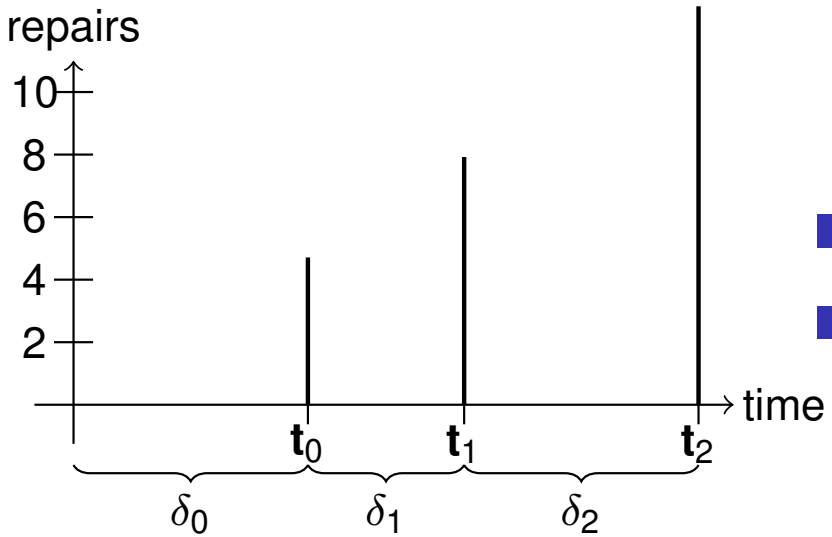

- analyses with δ_i instead of $\mathbf{t}_i$
- more efficient

(A) Properties: Solution method (1/3)

- abstract CTMC to ACTMC
  (special CTMDP, see later in (B))

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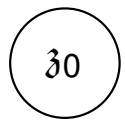

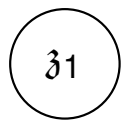

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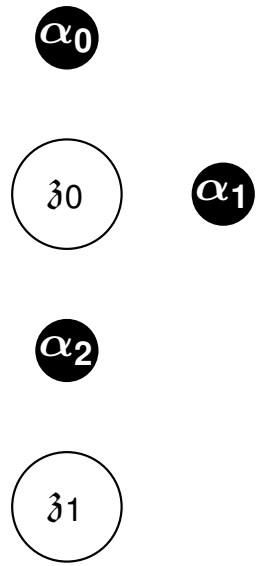

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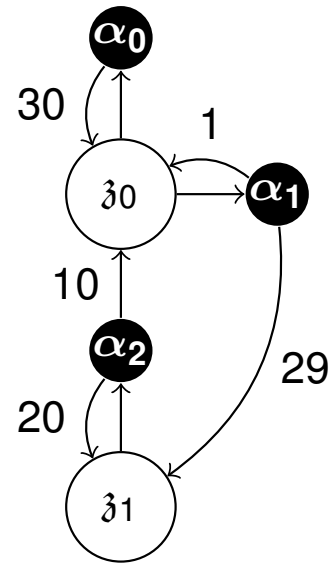

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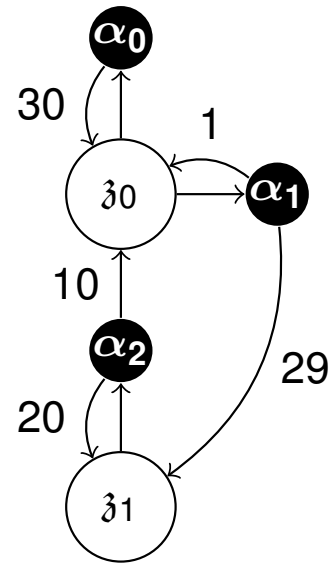

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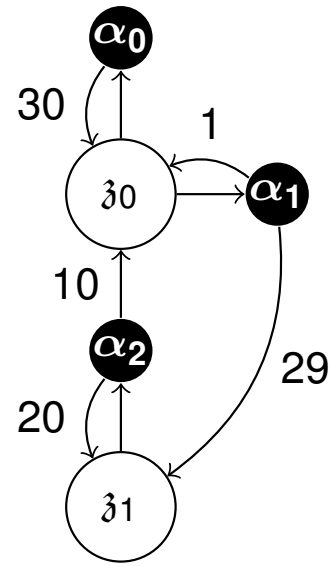

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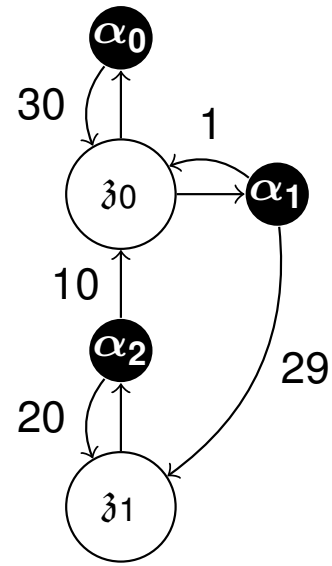

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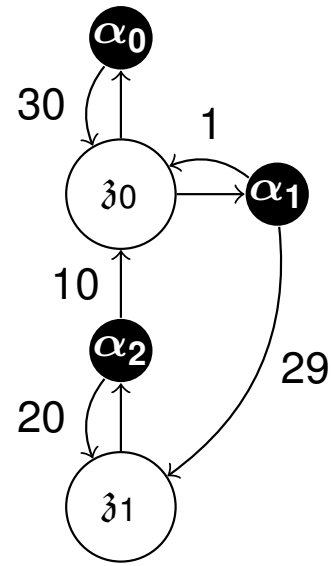
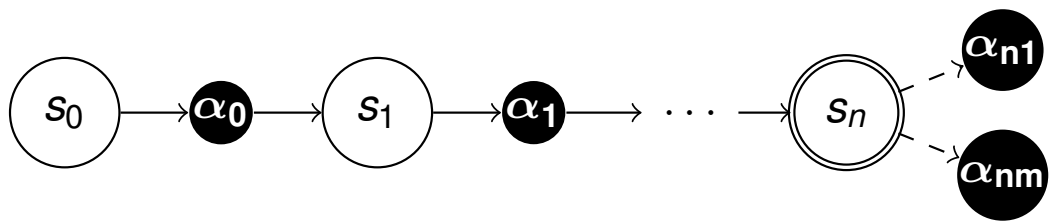
  - value of CTMDP depends on scheduler



(A) Properties: Solution method (2/3)

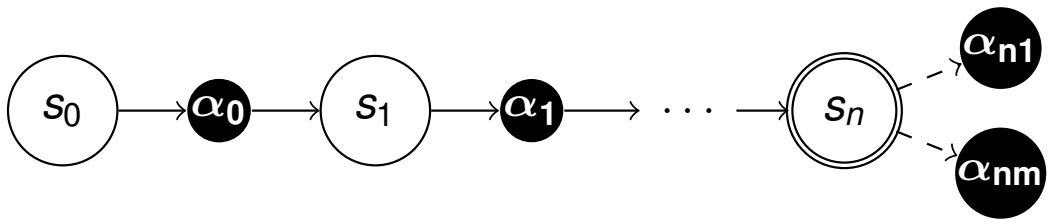
Compute maximal (minimal) value over σ which



(A) Properties: Solution method (2/3)

Compute maximal (minimal) value over σ which

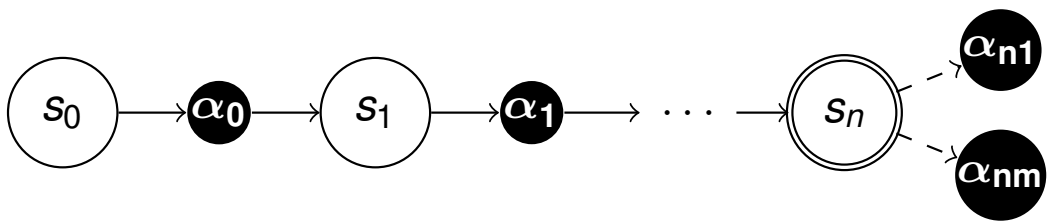
- can apply a probabilistic choice over actions



(A) Properties: Solution method (2/3)

Compute maximal (minimal) value over σ which

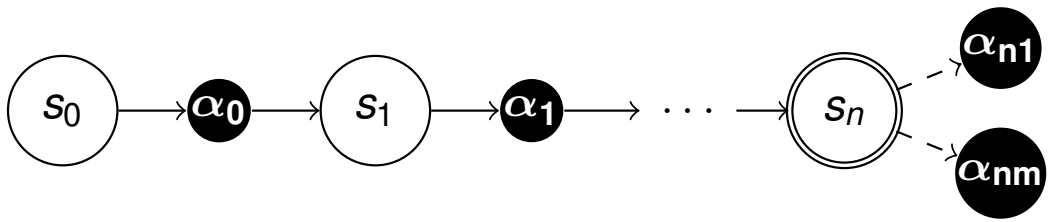
- can apply a probabilistic choice over actions
- know sequence of past actions and states



(A) Properties: Solution method (2/3)

Compute maximal (minimal) value over σ which

- can apply a probabilistic choice over actions
- know sequence of past actions and states
- does not know exact time points



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Compute maximal (minimal) value over σ which

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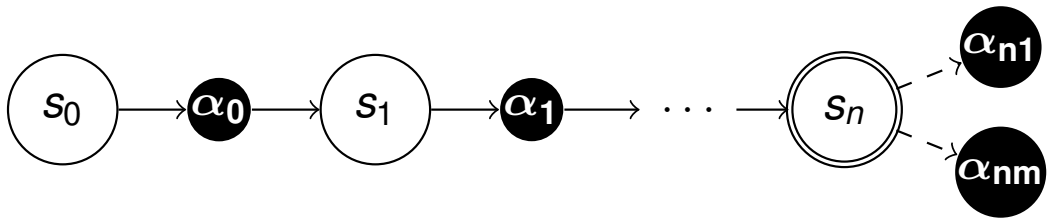
- can not change decision before leaving state



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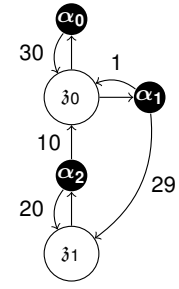

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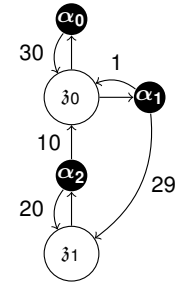

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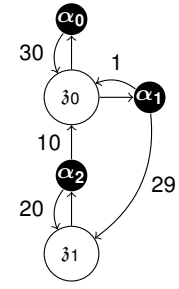

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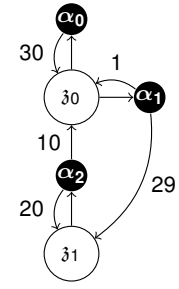

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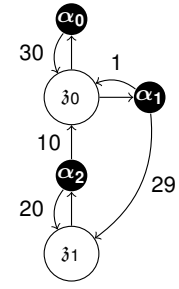

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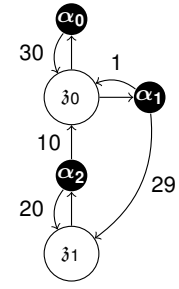

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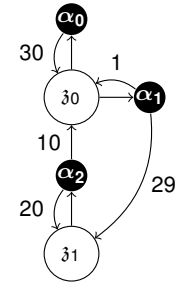

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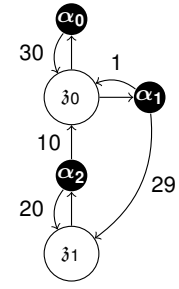

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- probabilistic choice over actions not needed

[1] C. Baier, H. Hermanns, J.-P. Katoen, and B. R. Haverkort, “Efficient computation of time-bounded reachability probabilities in uniform continuous-time Markov decision processes”,

[2] M. Kwiatkowska, G. Norman, and A. Pacheco, “Model checking expected time and expected reward formulae with random time bounds”

(A) Properties: Solution method (3/3)

Algorithm, based on [1] and [2]:

let k large enough, max. number jumps considered

$\phi(i)$ Poisson distr, $\psi(i) = \sum_{j>i} \phi(j)$ mixed Poisson

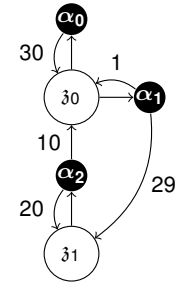
$\mathbf{P}(\cdot,\cdot) := \mathbf{R}(\cdot,\cdot)/\mathbf{u}$, $q_{k+1} := 0$

forall the $i = k, k-1, \ldots, 0$ **do**

  forall the $s \in S$ **do**

    $q_i(s) := \max_\alpha \sum_{s'} \mathbf{P}(s,\alpha,s') q_{i+1}(s')$

    $q_i(s) := q_i(s) \; + \; \phi_{\mathbf{ut}}(i) \cdot \mathbf{r}_f(s) \; + \; \psi_{\mathbf{ut}}(i) \cdot \mathbf{r}_c(s)/\mathbf{u}$



algorithm also shows that for maximising schedulers

- probabilistic choice over actions not needed
- history not needed, knowing step number suffices

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(B) Abstraction: ACTMCs

- ACTMC $\mathcal{C}$

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  - $S, \mathbf{u}, \mathbf{r}$: as for CTMCs and CTMDPs

$\mathfrak{Z}0$

$\mathfrak{Z}1$

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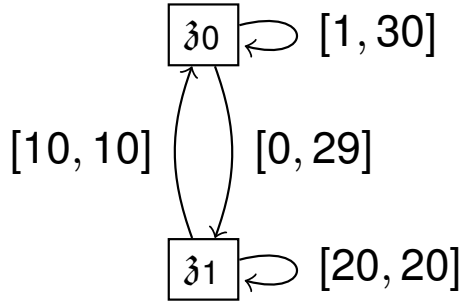
  - $\mathbf{l}^{\ell}, \mathbf{l}^{\mathbf{u}} : (S \times S) \to [0, \mathbf{u}]$



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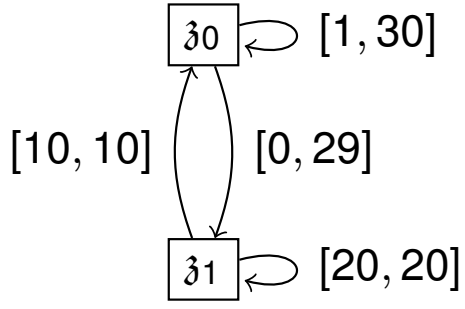

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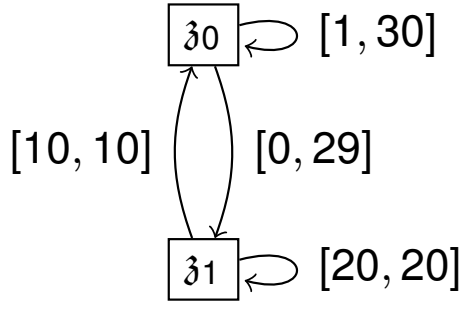

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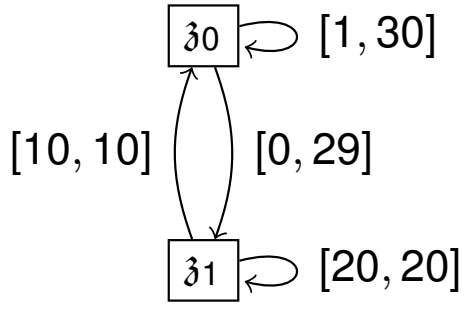

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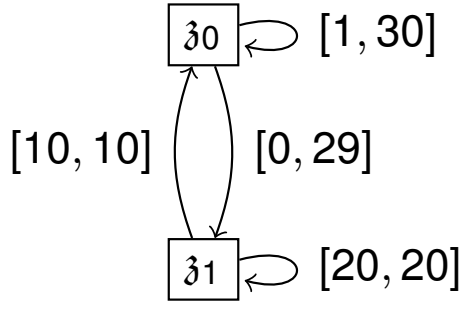
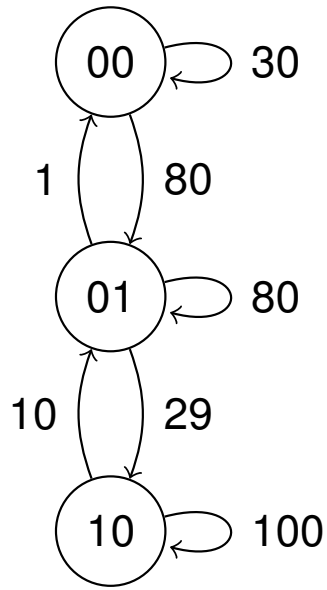
  - $|Act| = \infty$
  - but $\max_{\alpha} \ldots$ still computable efficiently



(B) Abstraction: State partitioning

- start with `PRISM` (guarded-command like) description

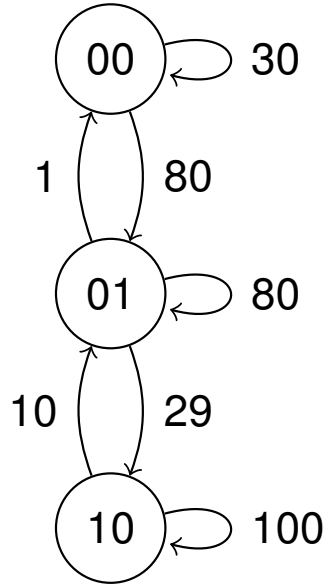
```
x0,x1 : bool init f;
...
[e]x0&!x1->29:x0'=f&x1'=f;
...
```



(B) Abstraction: State partitioning

- start with `PRISM` (guarded-command like) description
- remove rates

```
x0,x1 : bool init f;
...
[e]x0&!x1->x0'=f&x1'=t;
...
```



(B) Abstraction: State partitioning

- start with PRISM (guarded-command like) description
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- compute reachable states as BDD (e.g. NuSMV)

```
x0,x1 : bool init f;
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[e]x0&!x1->x0'=f&x1'=t;
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```
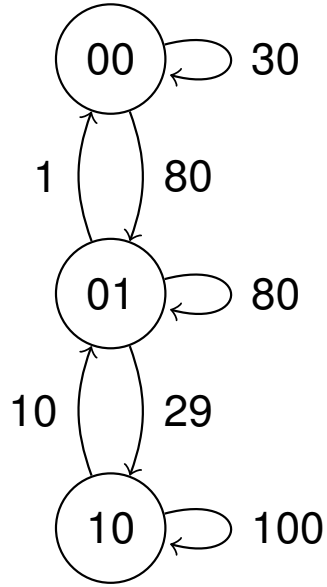
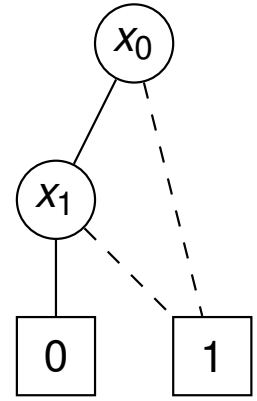
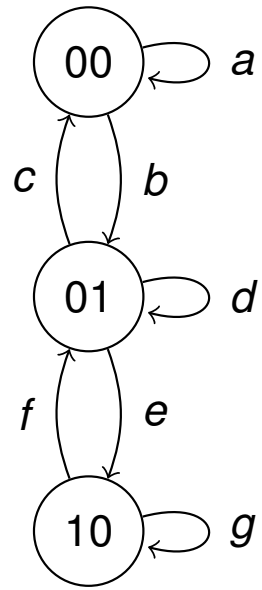



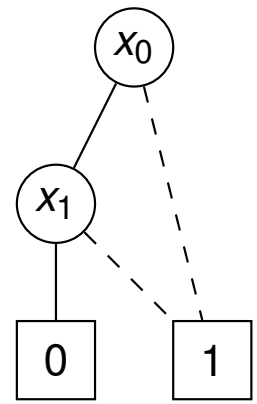


(B) Abstraction: State partitioning

- start with `PRISM` (guarded-command like) description
- remove rates
- compute reachable states as BDD (e.g. `NuSMV`)
- consider commands as labels *a b*, etc. of labelled transition system

```
x0,x1 : bool init f;
...
[e]x0&!x1->x0'=f&x1'=t;
...
```





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- start with `PRISM` (guarded-command like) description
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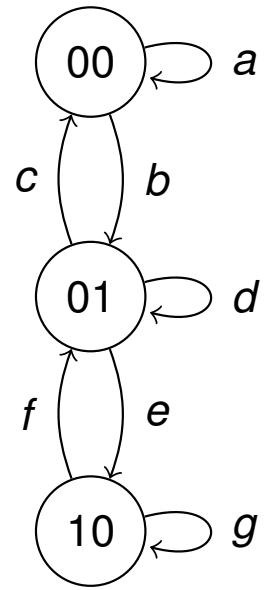
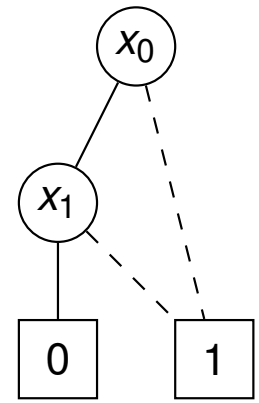
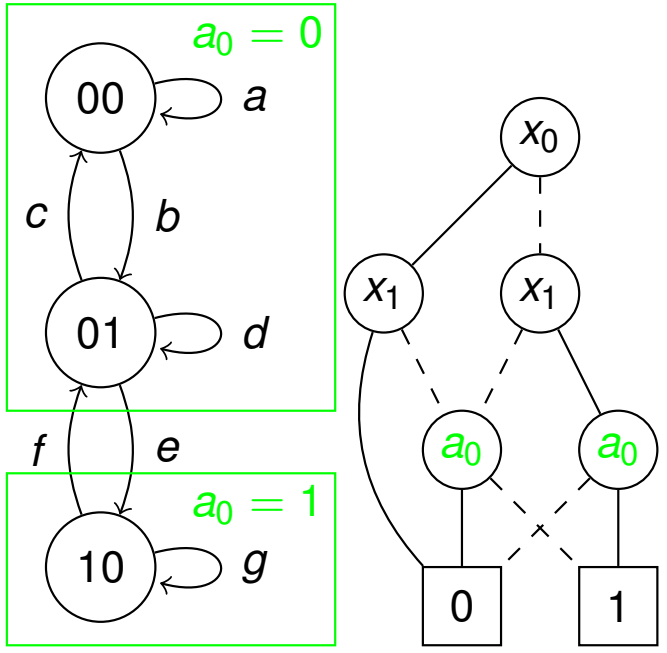




(B) Abstraction: State partitioning

- start with `PRISM` (guarded-command like) description
- remove rates
- compute reachable states as BDD (e.g. `NuSMV`)
- consider commands as labels *a b*, etc. of labelled transition system
- partitioning using symbolic `sig`nature `ref`inement
- by introducing additional BDD vars

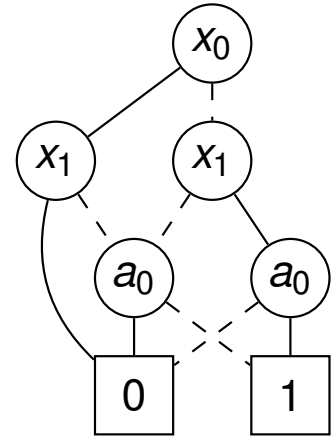
```
x0,x1 : bool init f;
...
[e]x0&!x1->x0'=f&x1'=t;
...
```



(B) Abstraction: Building ACTMCs

- prepare states of ACTMC

state partitioning:



PRISM model:

```
x0&!x1->
 29:x0'=f&x1'=t;
```

ACTMC to build:

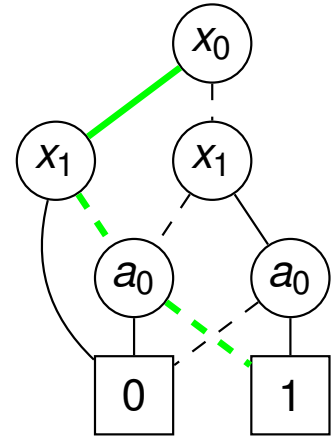
$a_0 = 0$

$a_0 = 1$

(B) Abstraction: Building ACTMCs

- prepare states of ACTMC
- consider each valid variable valuation

state partitioning:



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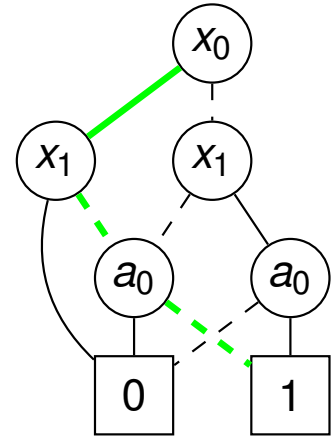
(B) Abstraction: Building ACTMCs

- prepare states of ACTMC
- consider each valid variable valuation
- get abstract state

state partitioning:



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- prepare states of ACTMC
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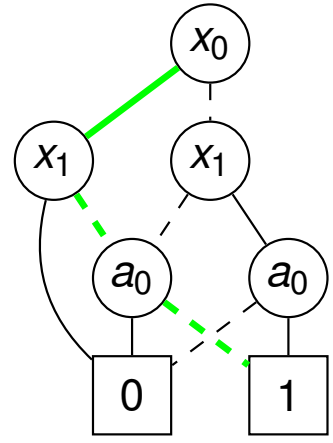
- get abstract state
- get contained concrete state

state partitioning:



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(B) Abstraction: Building ACTMCs

- prepare states of ACTMC
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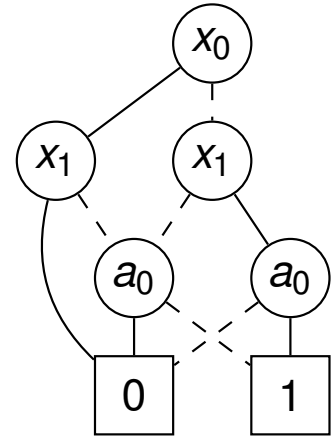
- get abstract state
- get contained concrete state
- `PRISM` model to get successor rates and states

state partitioning:



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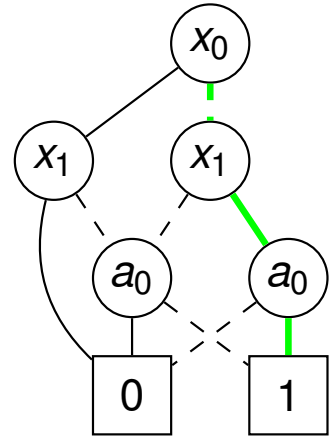

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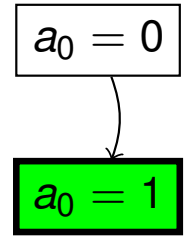
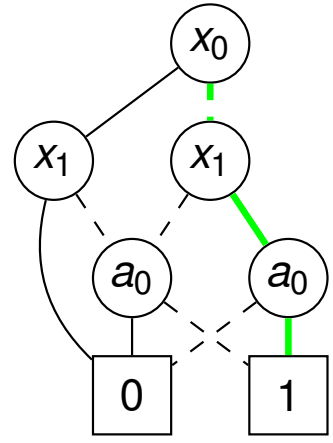


(B) Abstraction: Building ACTMCs

- prepare states of ACTMC
- consider each valid variable valuation
- get abstract state
- get contained concrete state
- PRISM model to get successor rates and states
- partitioning to find containing abstract states
- widen rates of ACTMC

state partitioning:



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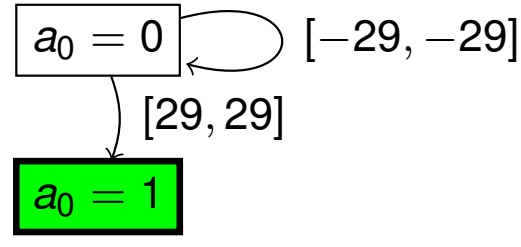

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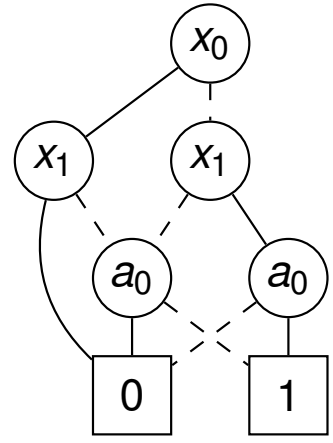

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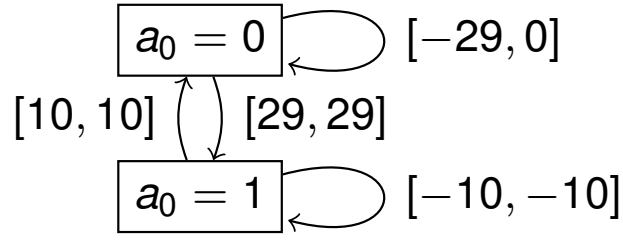

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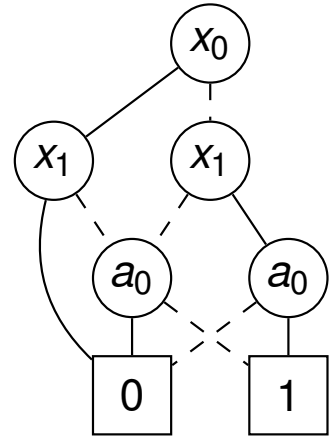
- widen rates of ACTMC
- rewards similar, but simpler

state partitioning:



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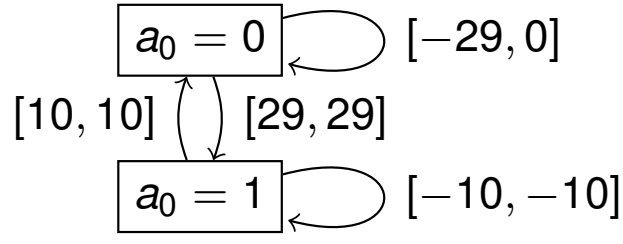

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- get contained concrete state
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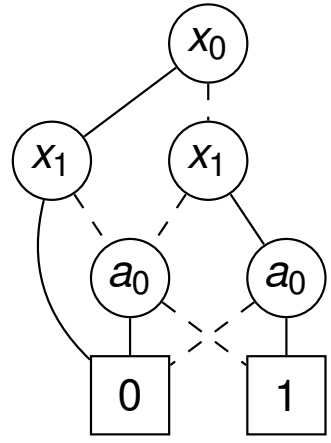
- widen rates of ACTMC
- rewards similar, but simpler
- finally, need some fixes

state partitioning:



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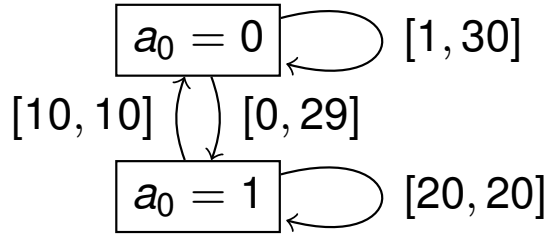
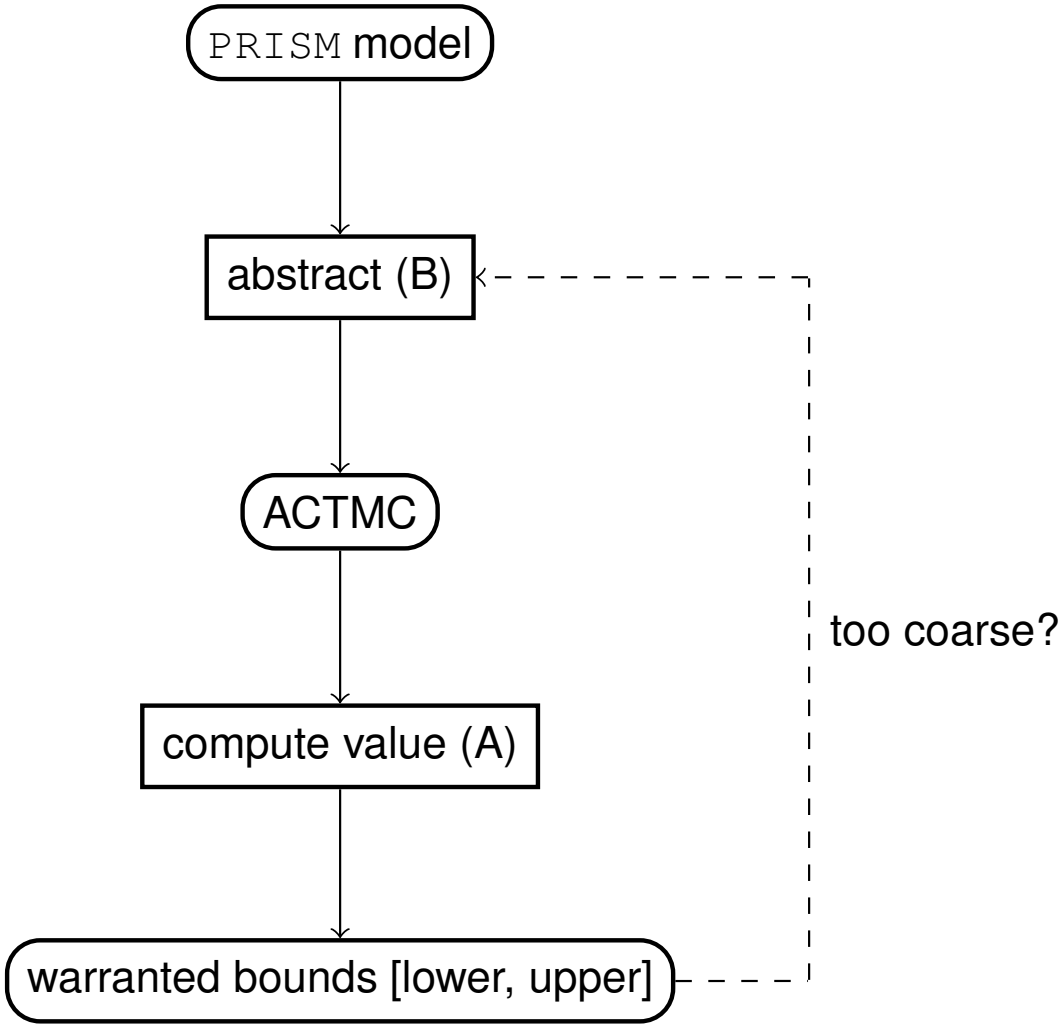


Overall scheme

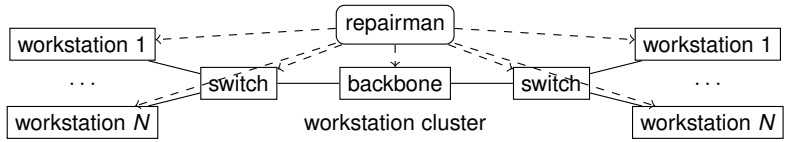


Case studies

- implementation of basic technique
- applied on

Case studies

- implementation of basic technique
- applied on
  - Workstation cluster



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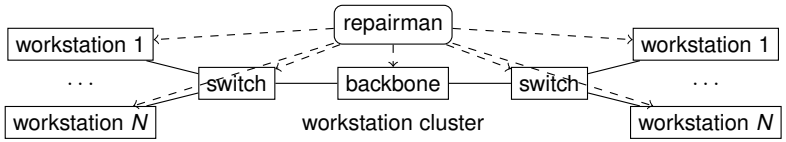

  - Google File System

Case studies

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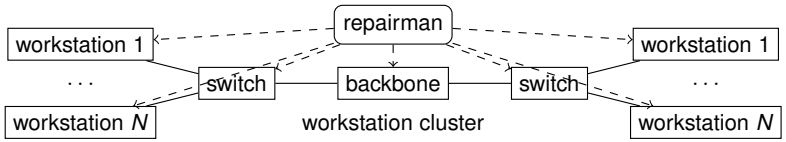

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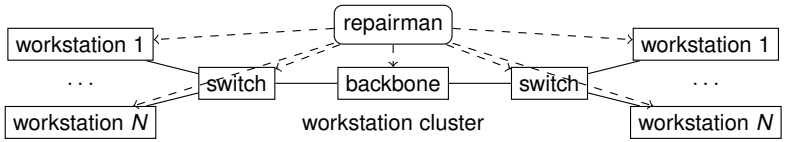

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- results promising, e.g. for cluster

		PRISM		A(E)CTMC Results			
N	$\|S\|$	T(m)	MB	$\|\mathfrak{A}\|$	T(m)	MB	Interval
64	$1.51 \cdot 10^5$	1	42	19420	2	89	$[127.98, 128.49]$
1024	$3.78 \cdot 10^7$	425	1015	19420	6	110	$[905.02, 905.20]$
8192	$2.42 \cdot 10^9$	**– Time out –**		19420	484	134	$[906.04, 906.04]$

Conclusion

- done so far
  - developed method to optimise general transient properties over relevant class of schedulers of CTMDPs
  - used for CTMC abstraction
  - discussed how to speed up abstraction
  - applied method on two case studies
- possible future works includes
  - improve refinement
  - abstraction of models already involving nondeterminism
  - integration into larger model checking context