Game-based Abstraction and Controller Synthesis for Probabilistic Hybrid Systems

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joint work with

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David Parker, University of Oxford, UK
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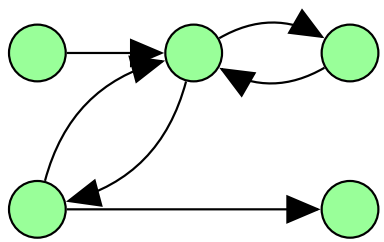
Lijun Zhang, Technical University of Denmark

ROCKS, October 5th, 2011

Motivation (1/2)

- many systems have discrete



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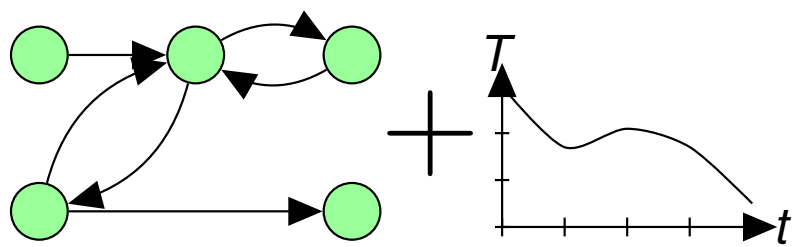
- and continuous behaviour



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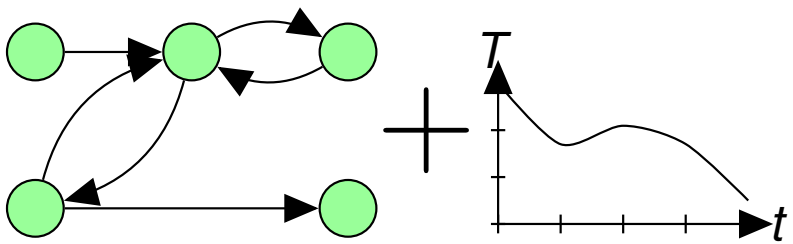
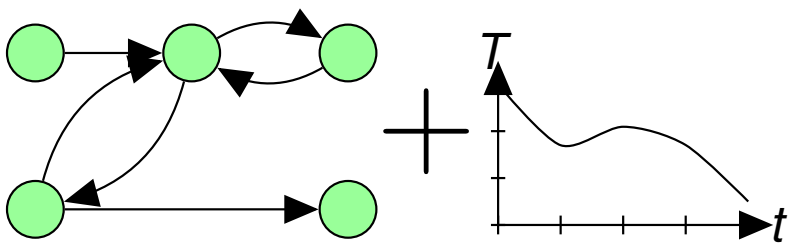


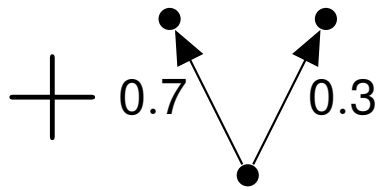
Motivation (1/2)

- many systems have discrete
- and continuous behaviour
- $\Rightarrow$ hybrid automata (handled by PHAVer, etc.)



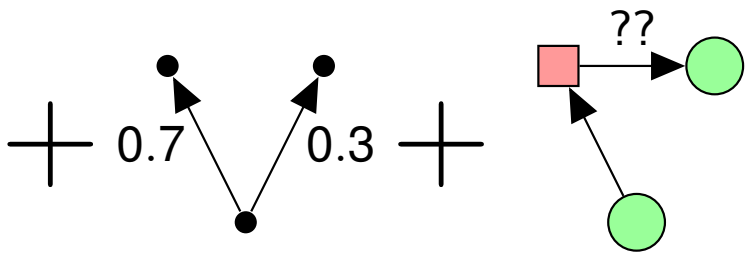
Motivation (2/2)

- often also probabilistic aspects (failures, etc.)



Motivation (2/2)

- often also probabilistic aspects (failures, etc.)
- only partially controllable



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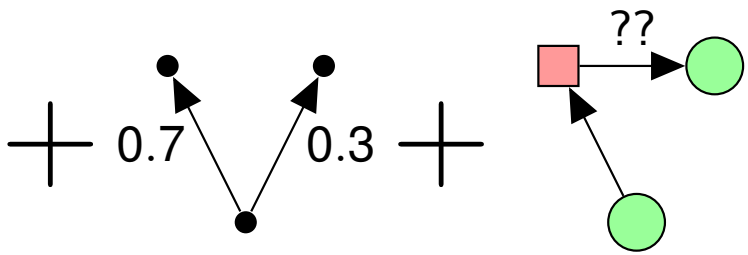
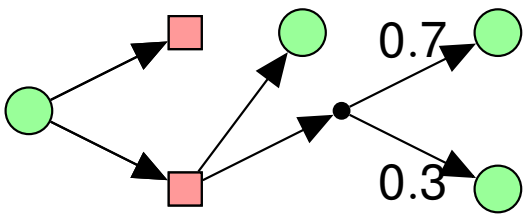


- $\Rightarrow$ game-based semantics for probabilistic hybrid systems



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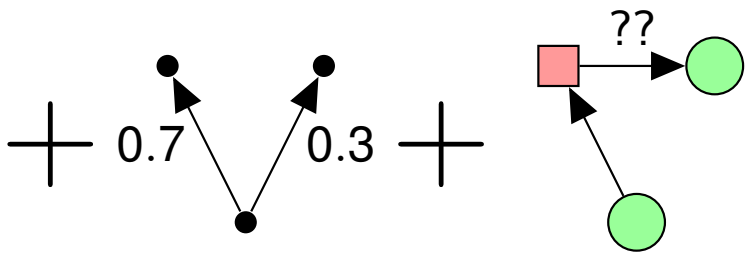
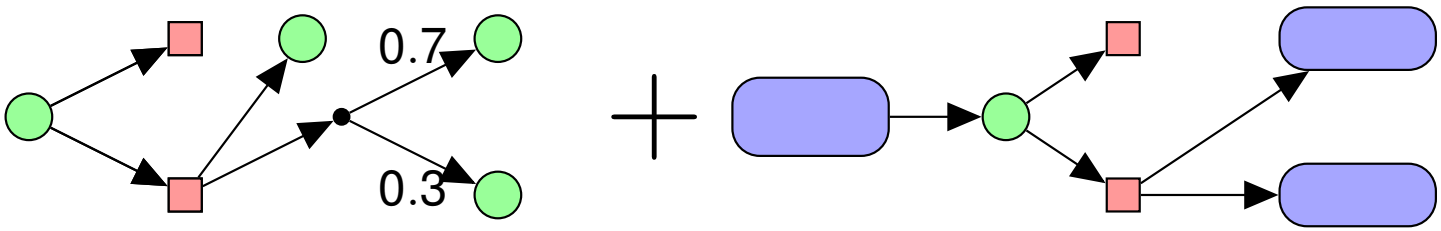
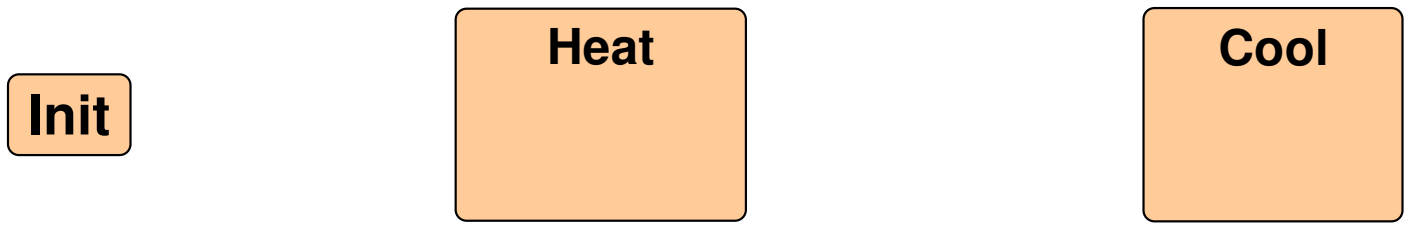


- $\Rightarrow$ game-based semantics for probabilistic hybrid systems
- $\Rightarrow$ finite abstractions to compute results



Hybrid Automata

- finite set of modes



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- finite set of modes
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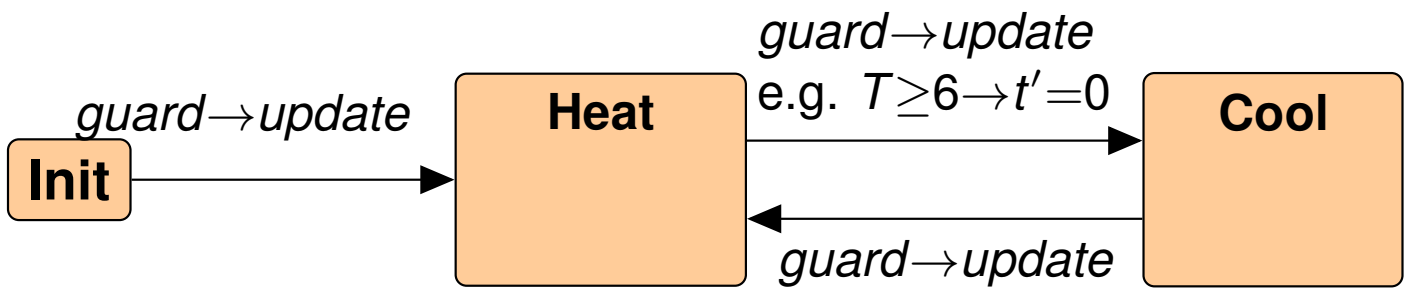

Hybrid Automata

- finite set of modes
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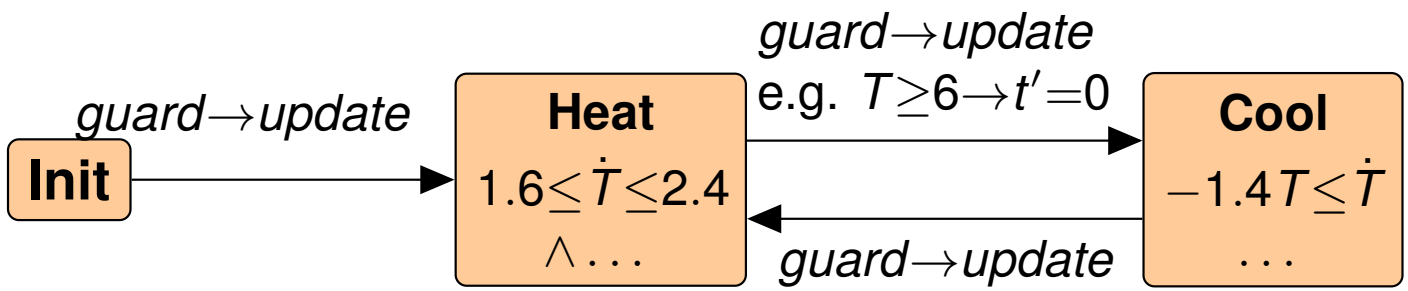
- continuous behaviour in modes



Probabilistic Hybrid Automata

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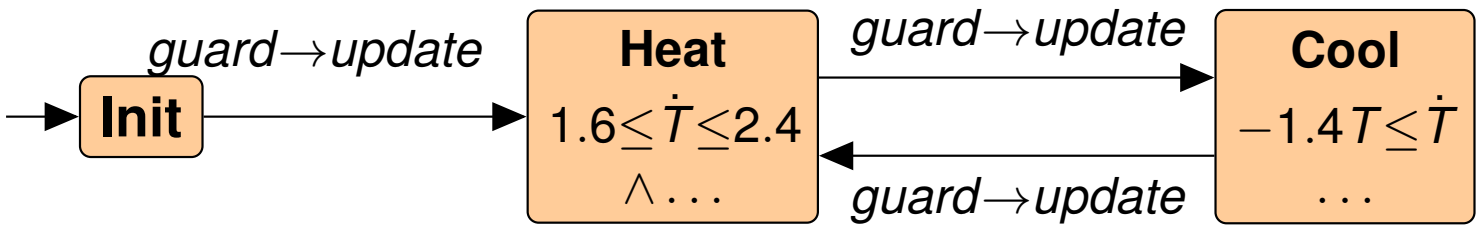
- continuous variables and behaviour in modes



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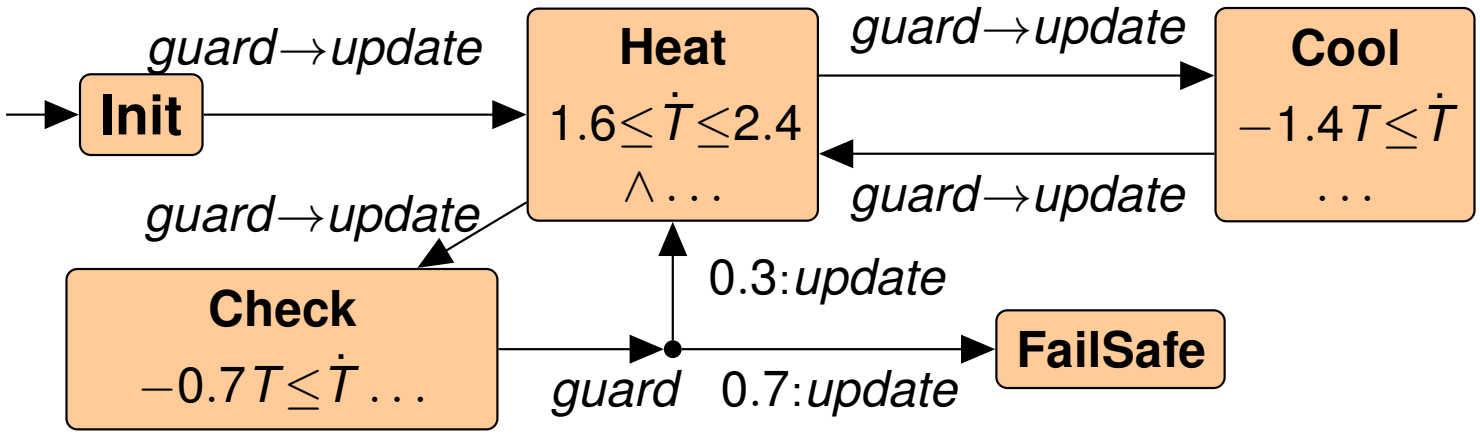

- probabilistic jump targets

Discrete Behaviour

- controller chooses command to execute

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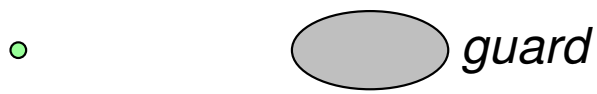

Discrete Behaviour

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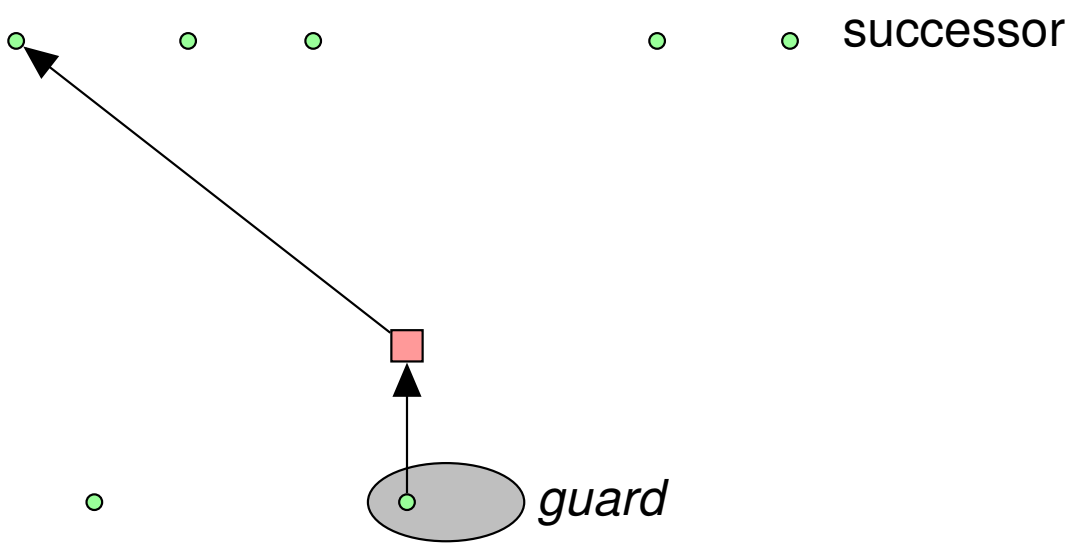
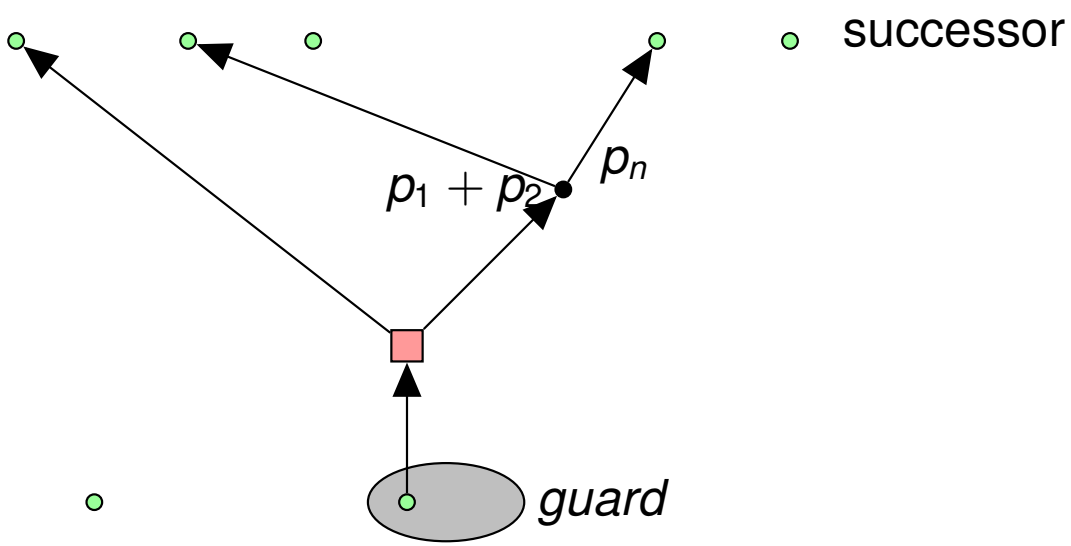


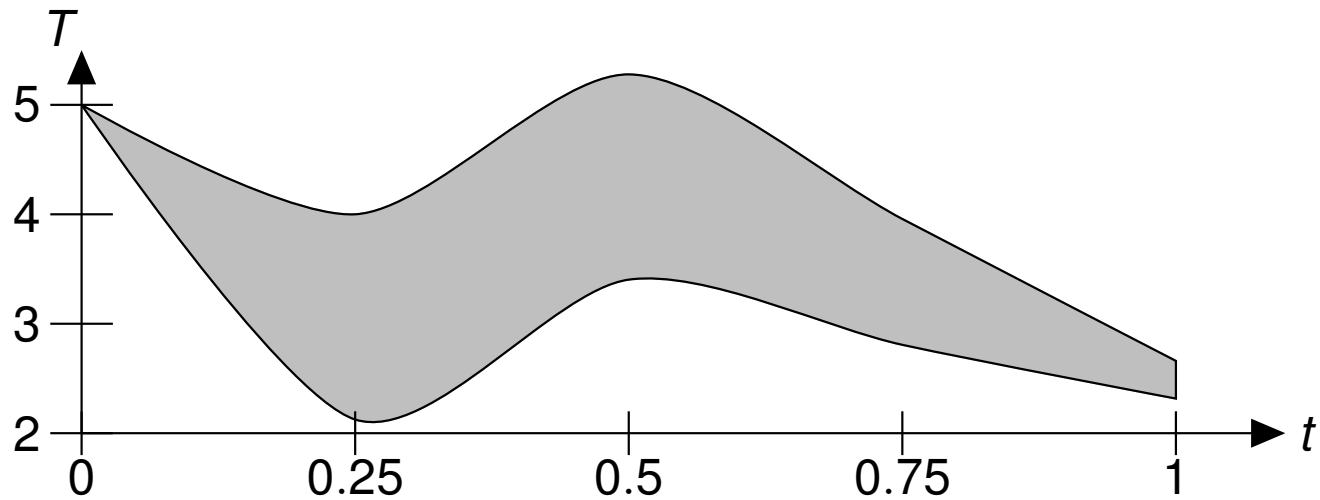
Discrete Behaviour

- controller chooses command to execute
- if state does not fulfill guard cannot execute
- else environment chooses exact effect
- possibly also probabilistic choice



Continuous Behaviour

- described by **differential inequation**



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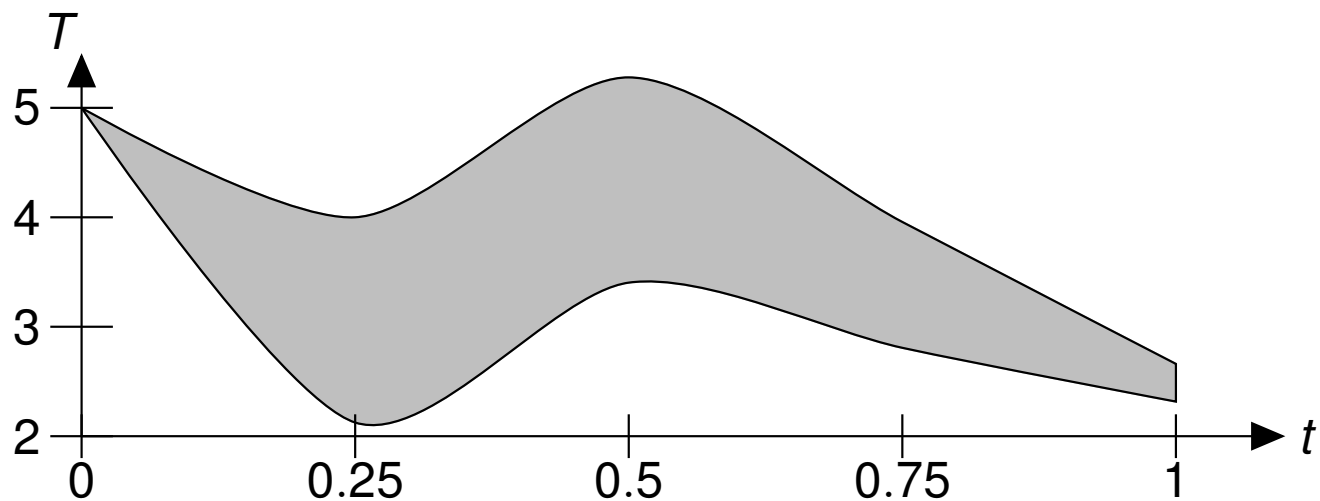

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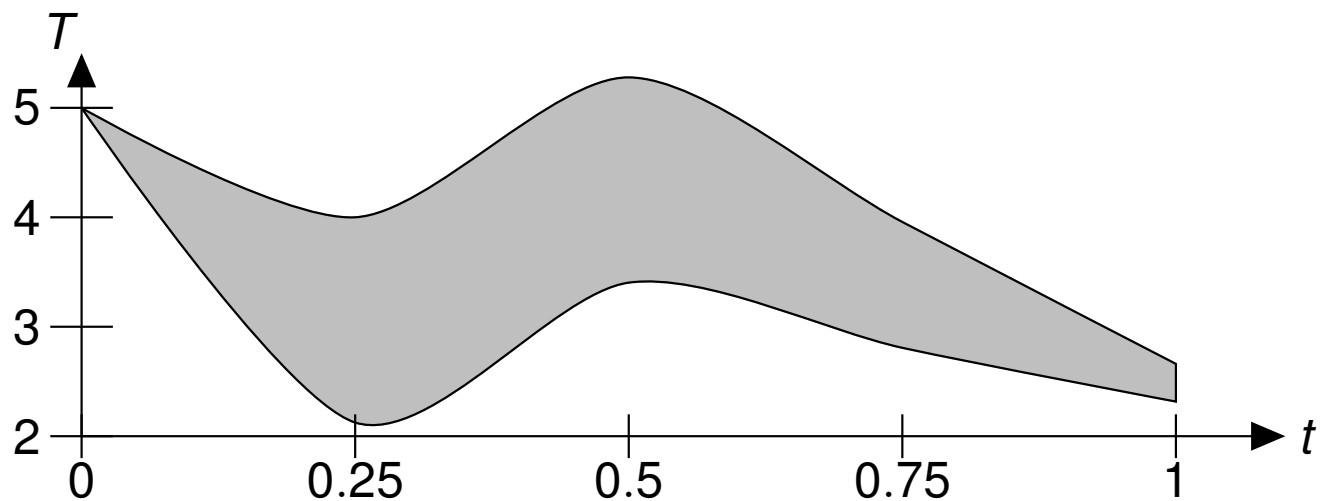

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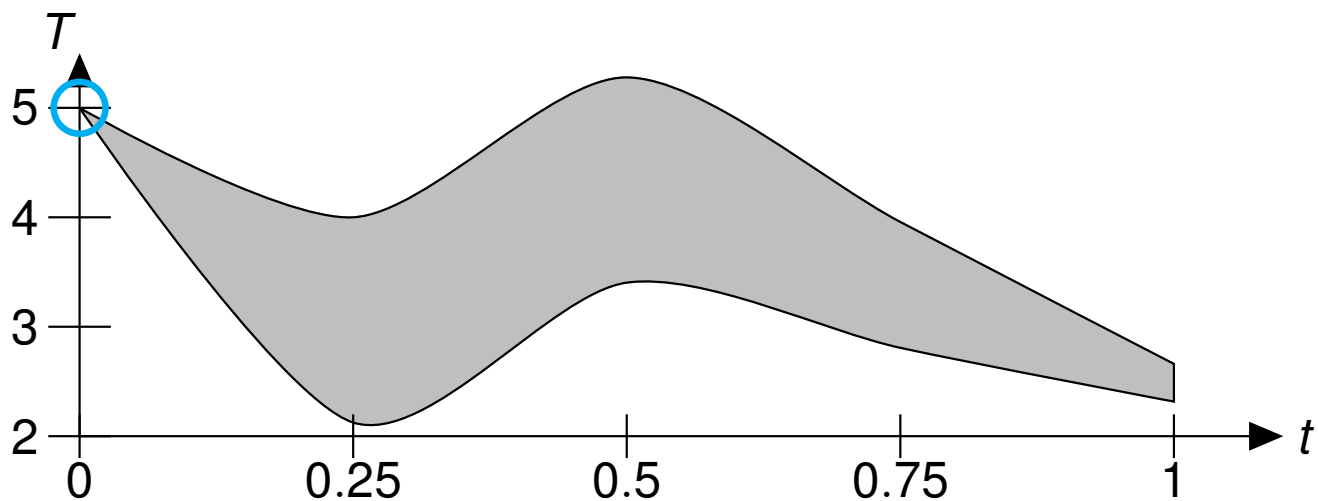

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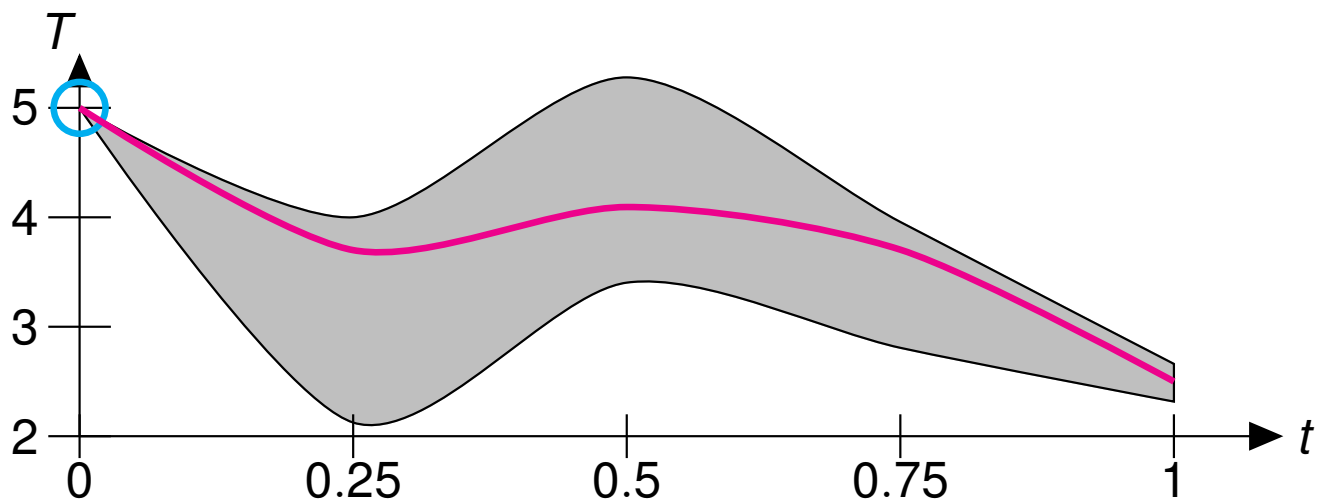

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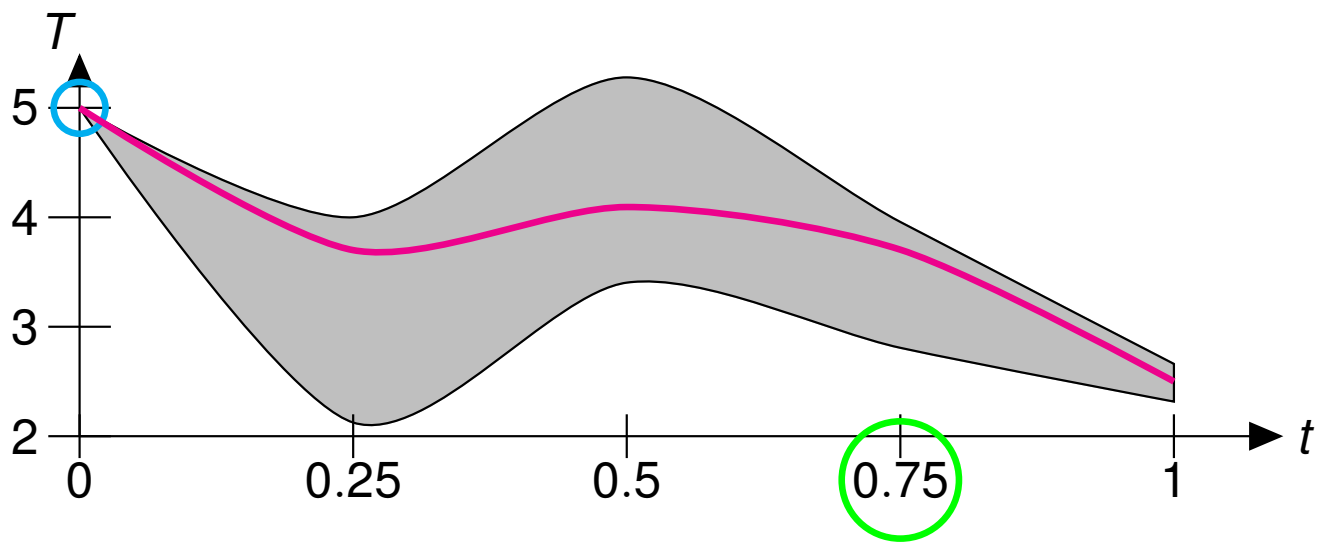

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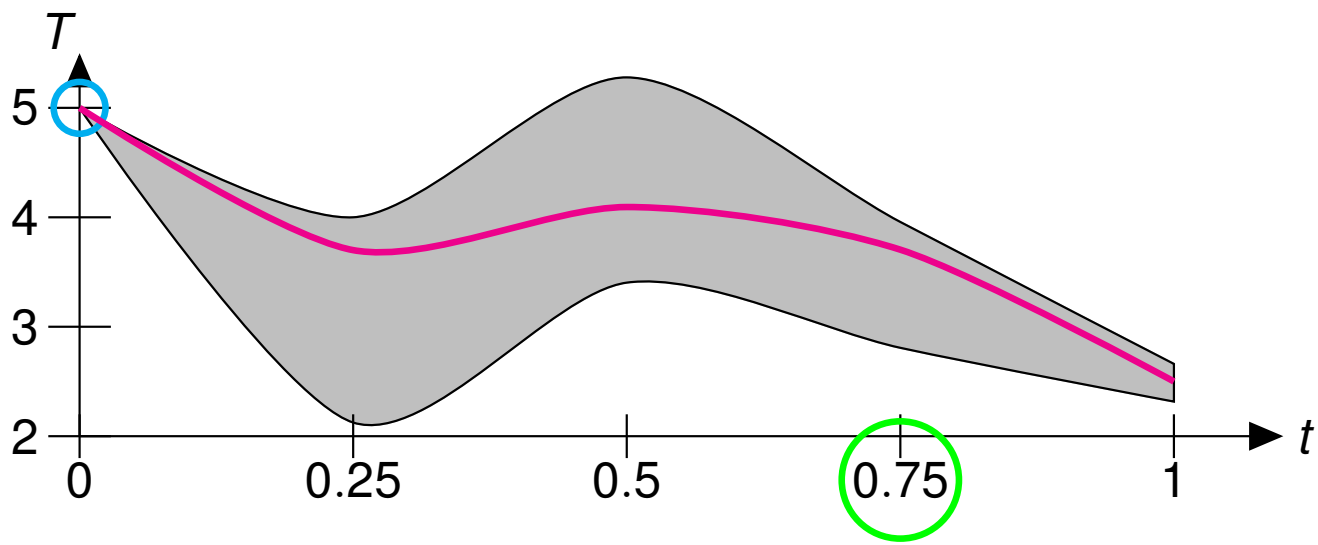

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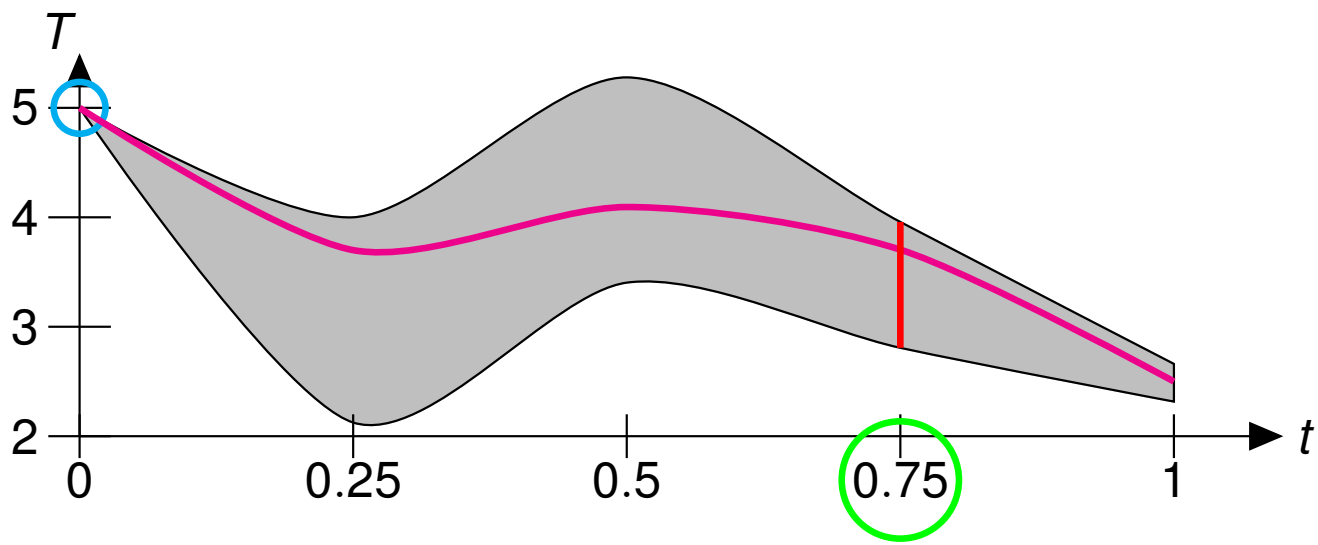
- effect given by $x(t)$



Semantics

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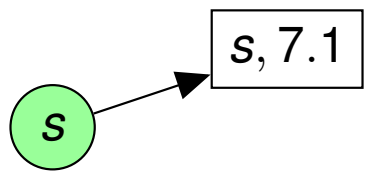
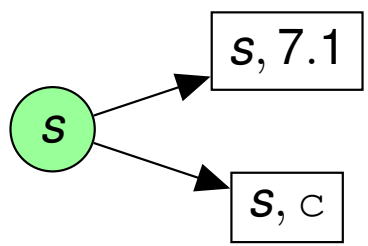
  - chooses whether to wait, and how long



Semantics

- controller
  - chooses whether to wait, and how long
  - or chooses which command to execute



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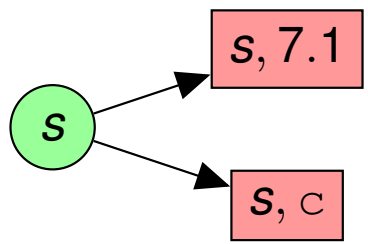

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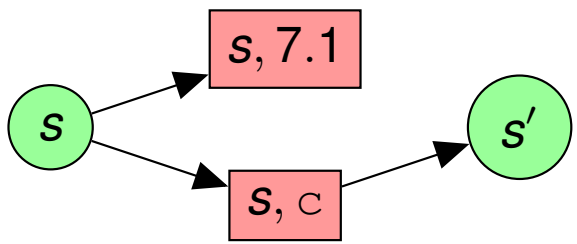

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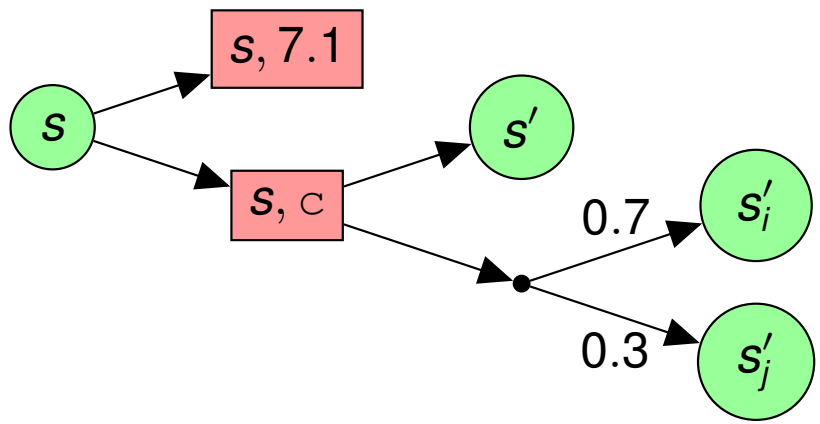

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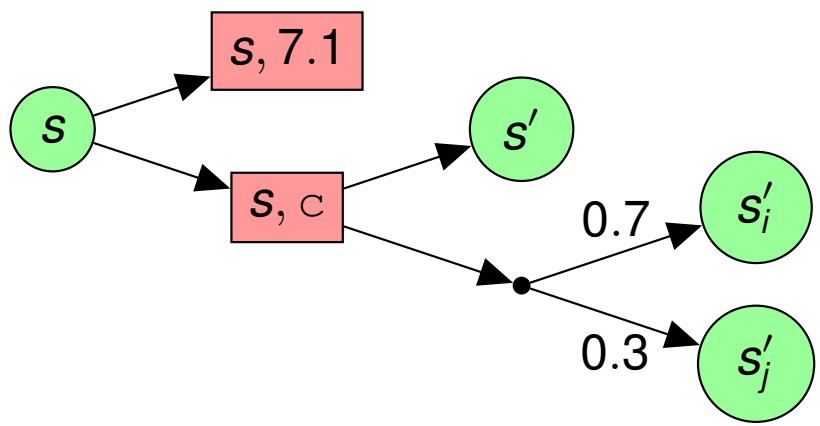
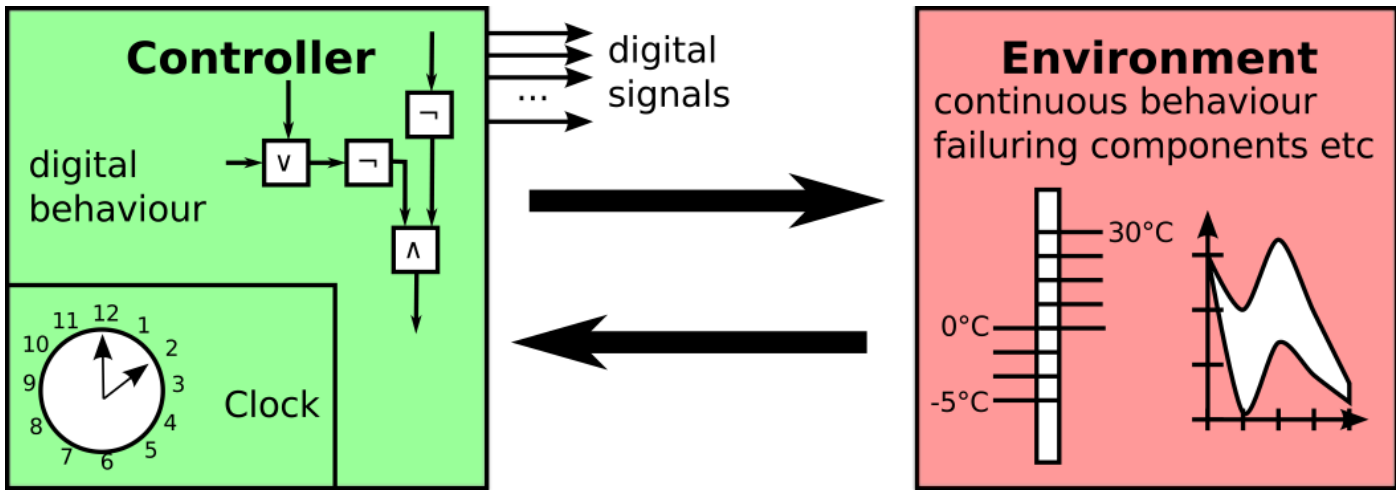
- probabilistic choice
- $\Rightarrow$ infinite probabilistic two-player game



Why Exactly This Semantics?

- natural



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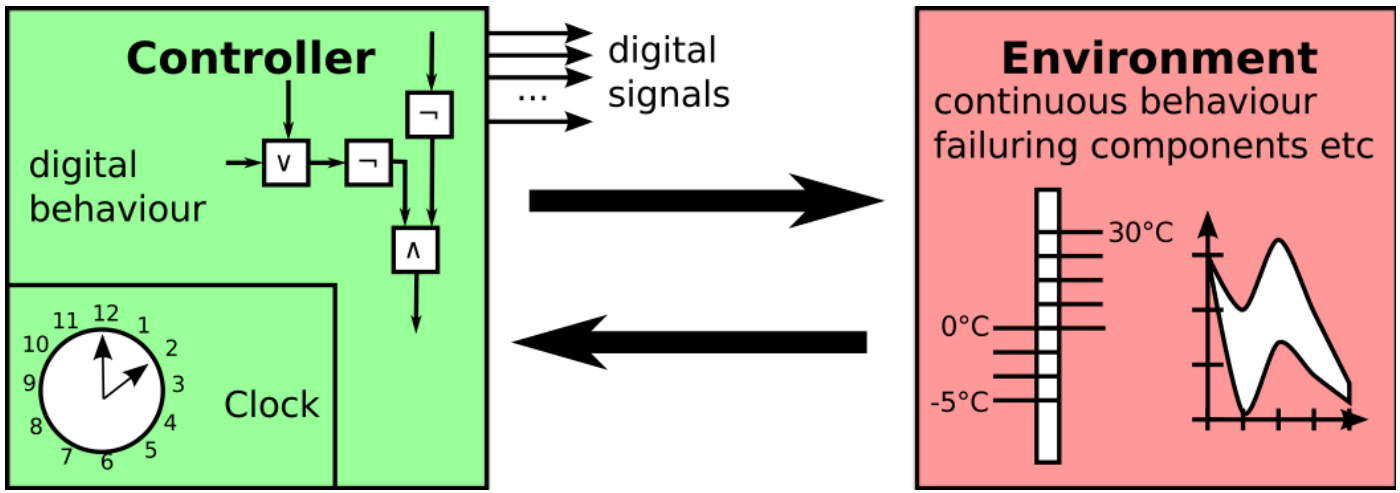

- fits to existing tools

...

HSolver **PHAVer**

...

SpaceEx

d/dt

... ...

Properties

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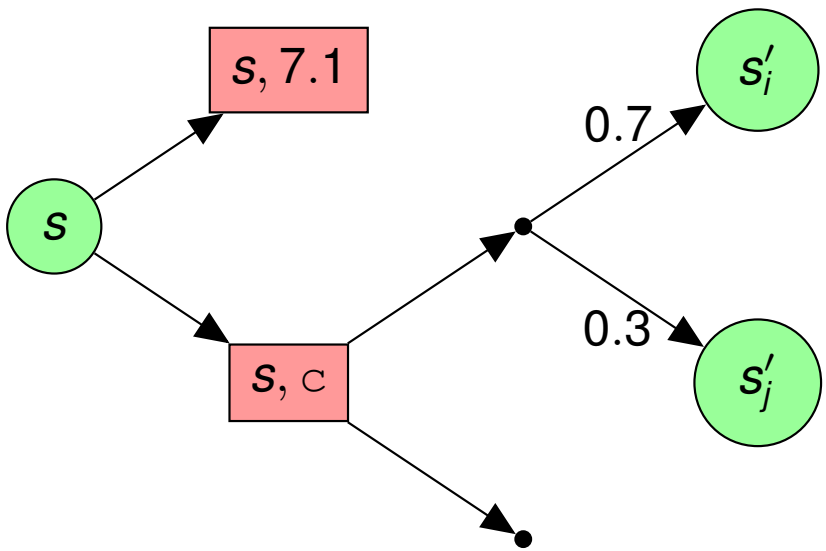

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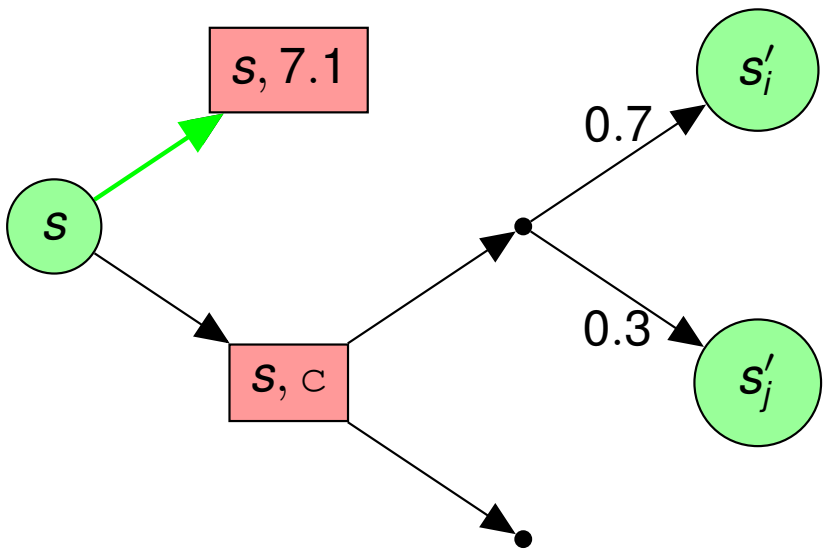

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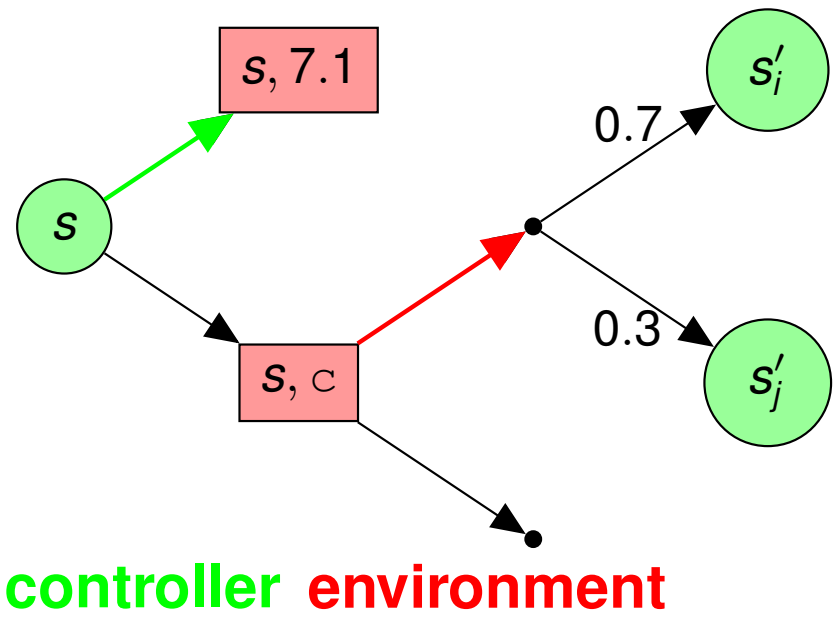

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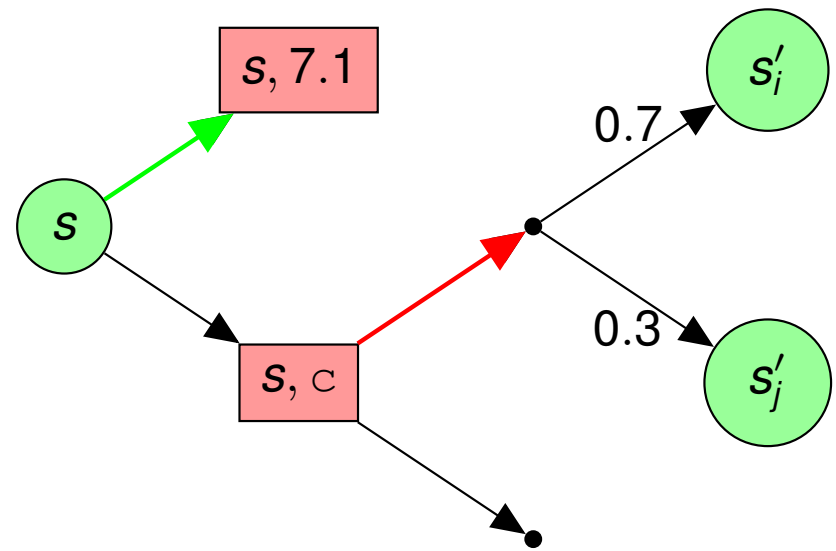
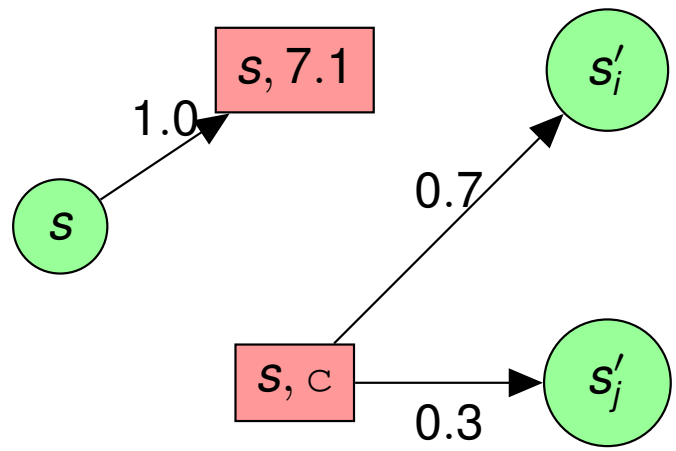


controller **environment**



stochastic process

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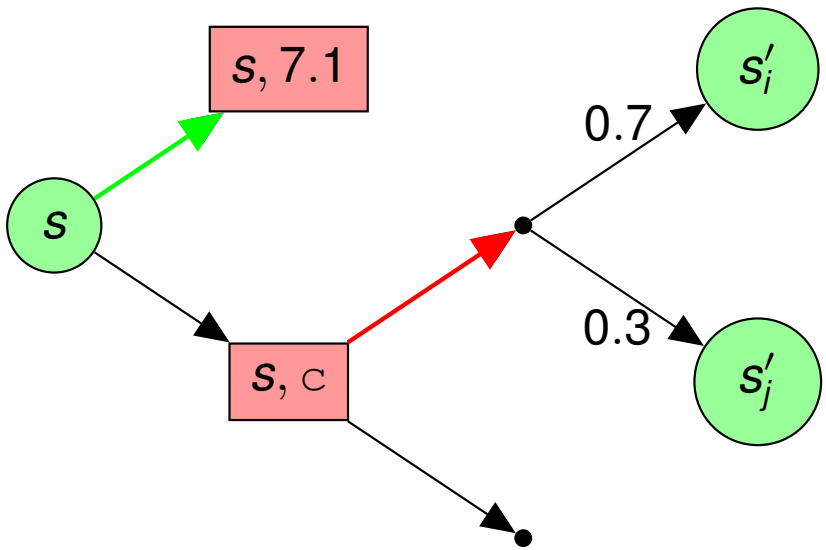

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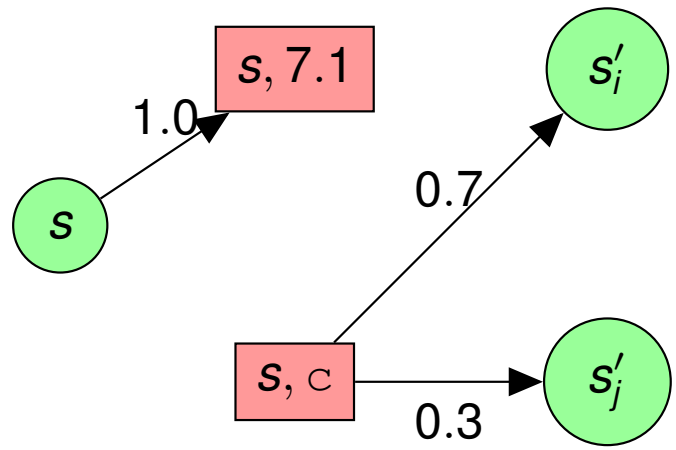

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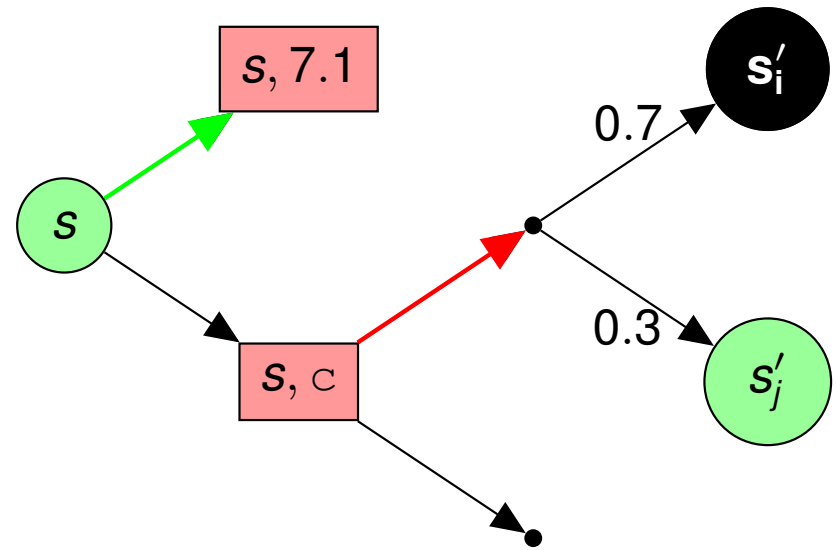

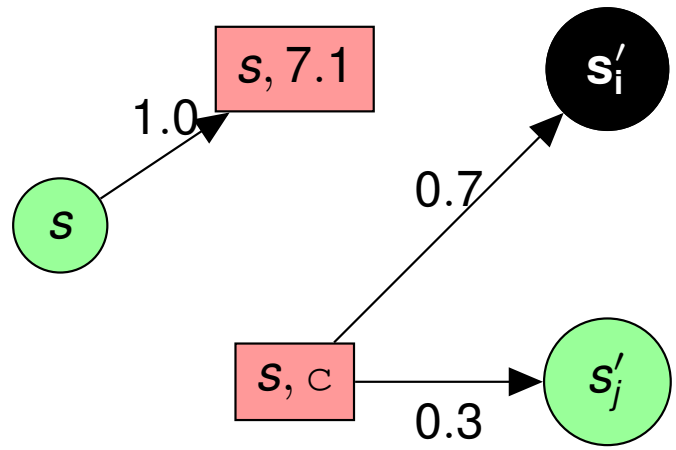

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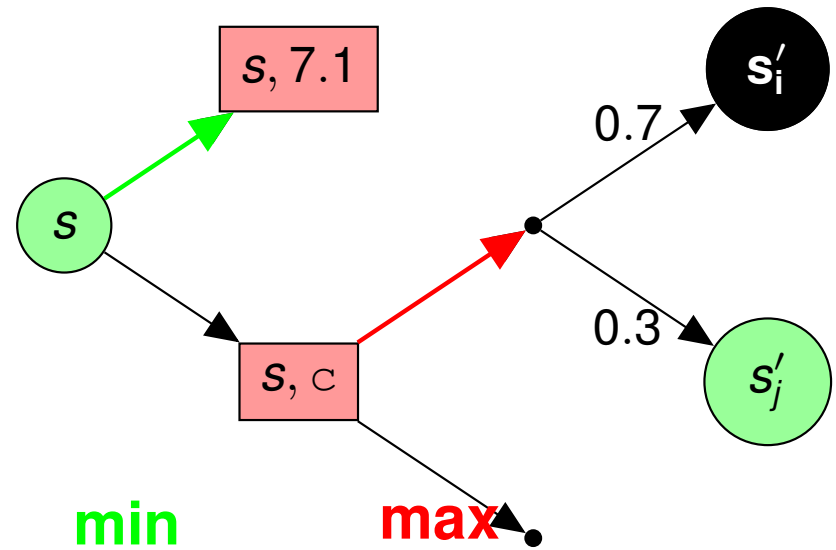

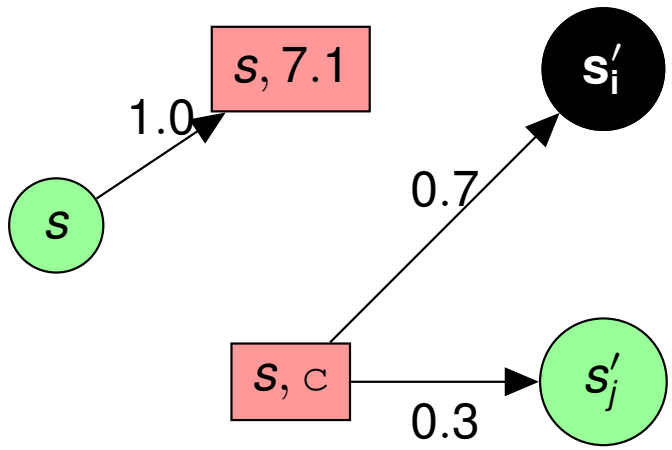

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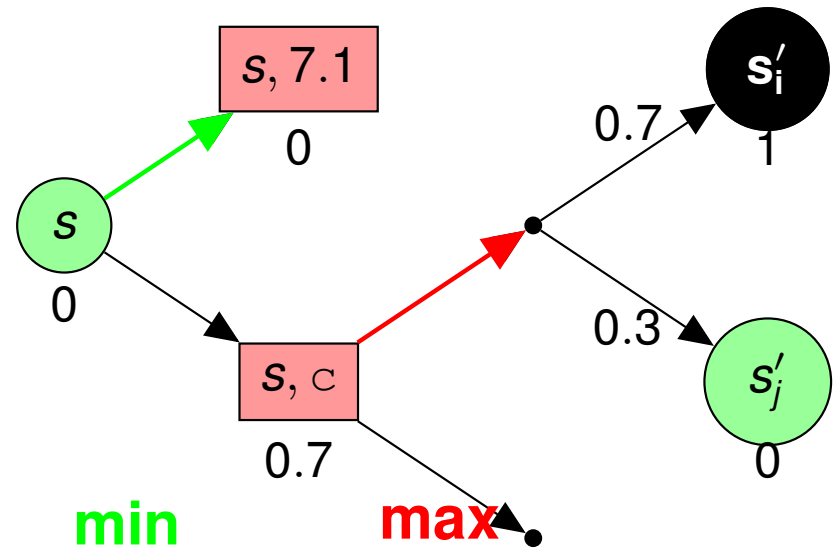
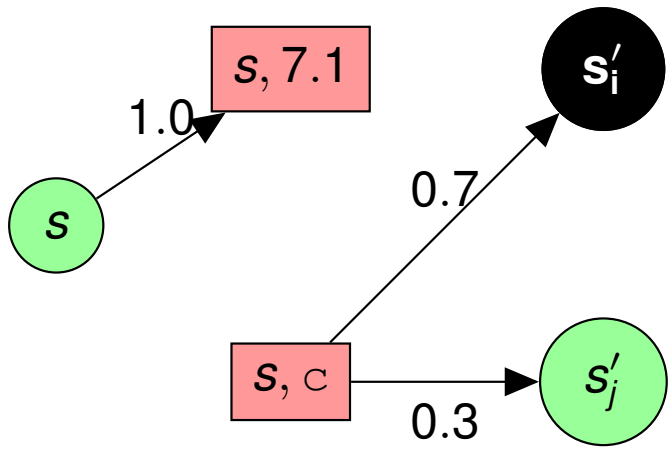
- **optimal value**: from optimal strategies objectives





stochastic process

Abstract State Space

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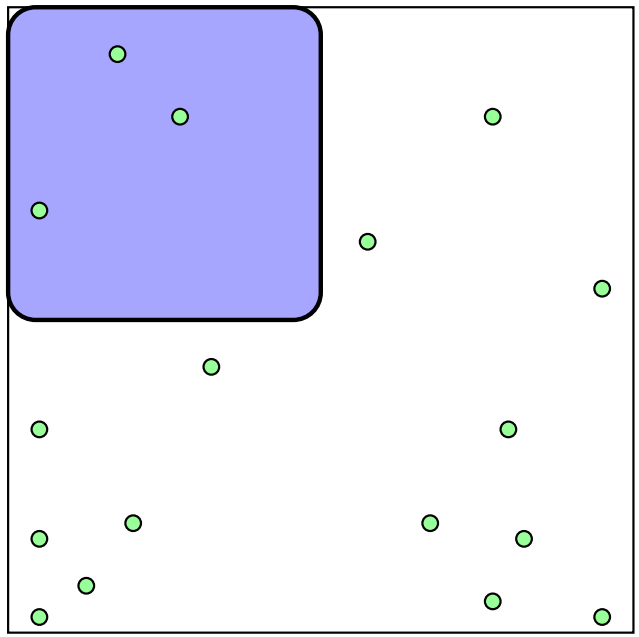

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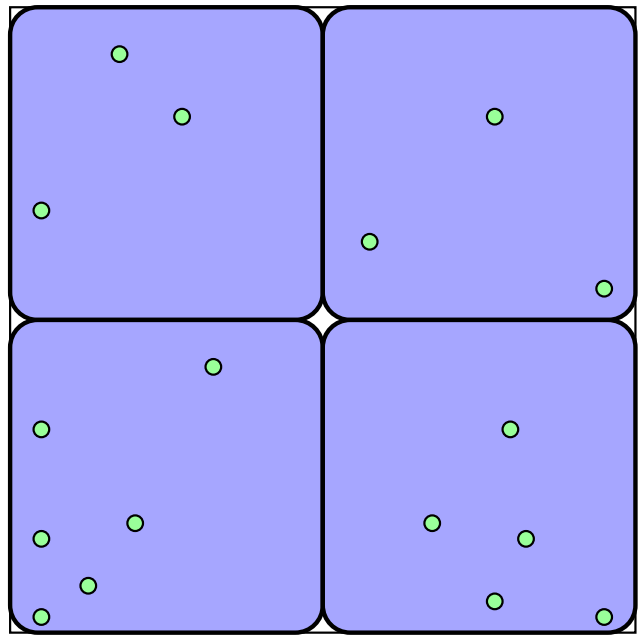
- **abstract state space:** set of abstract states which contains complete state space



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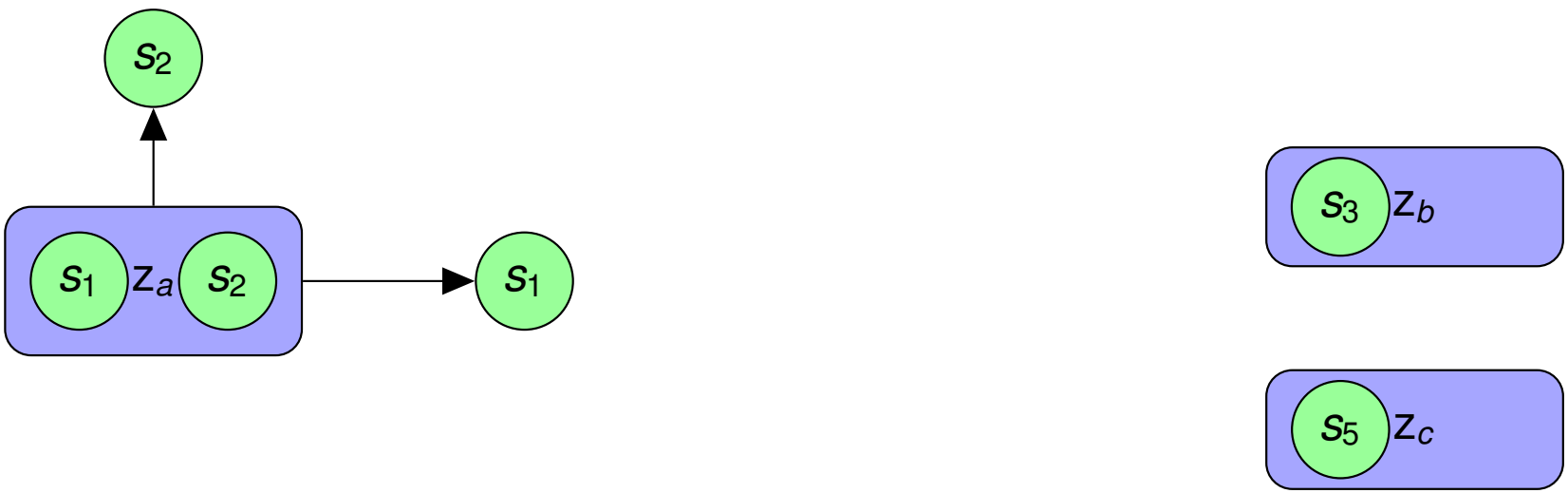

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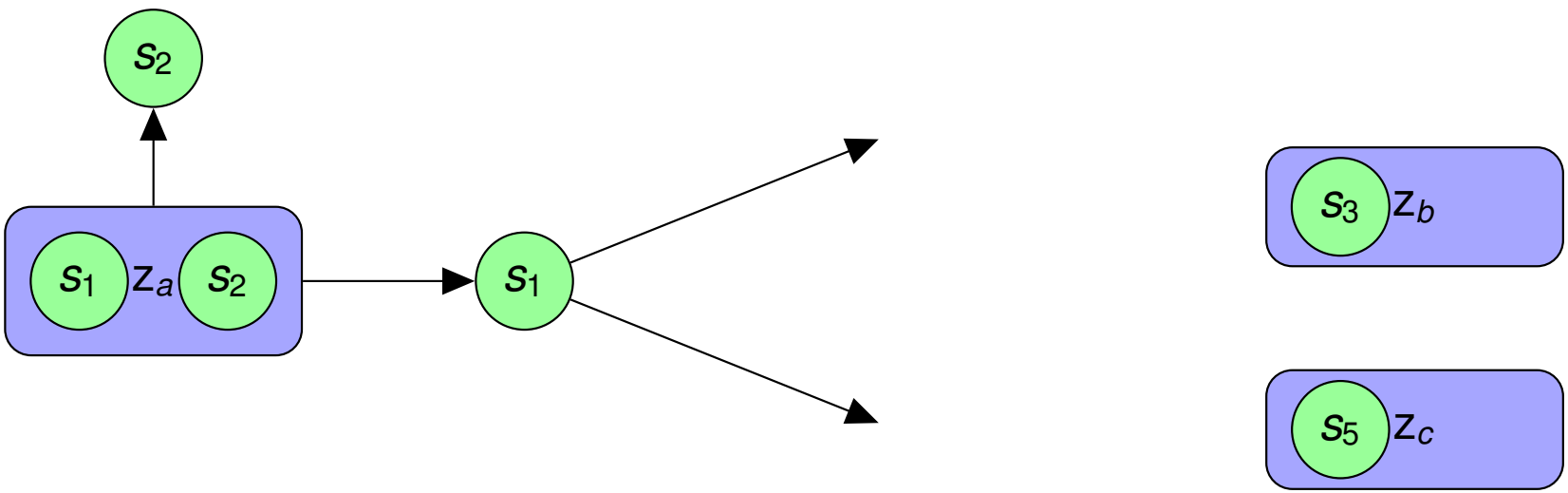

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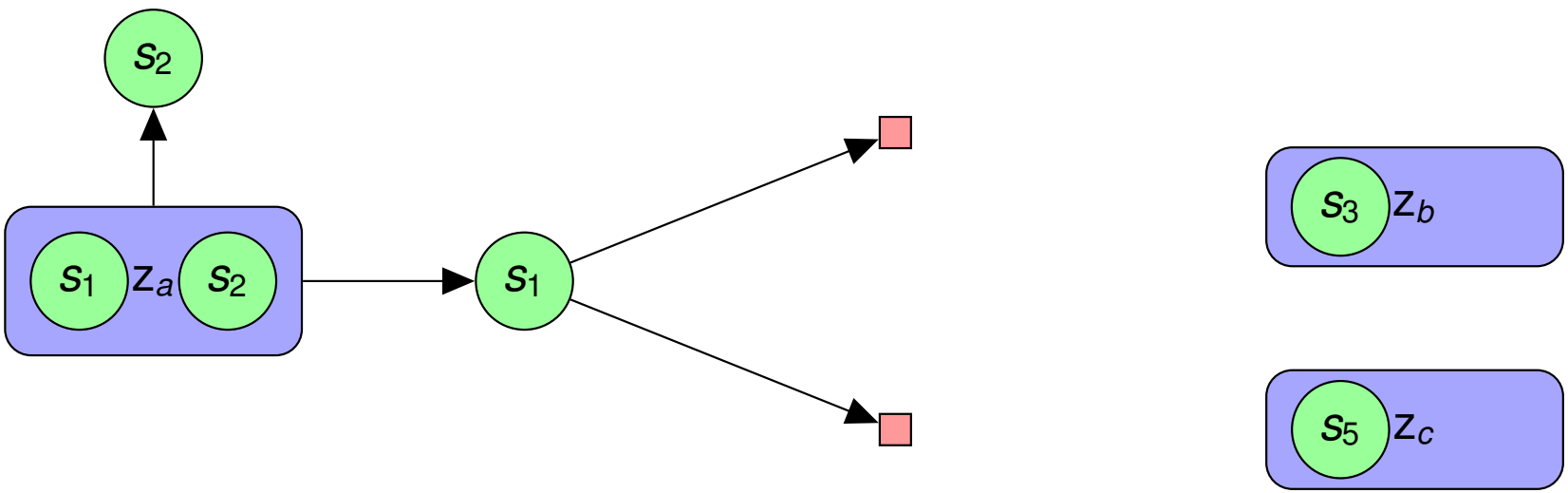

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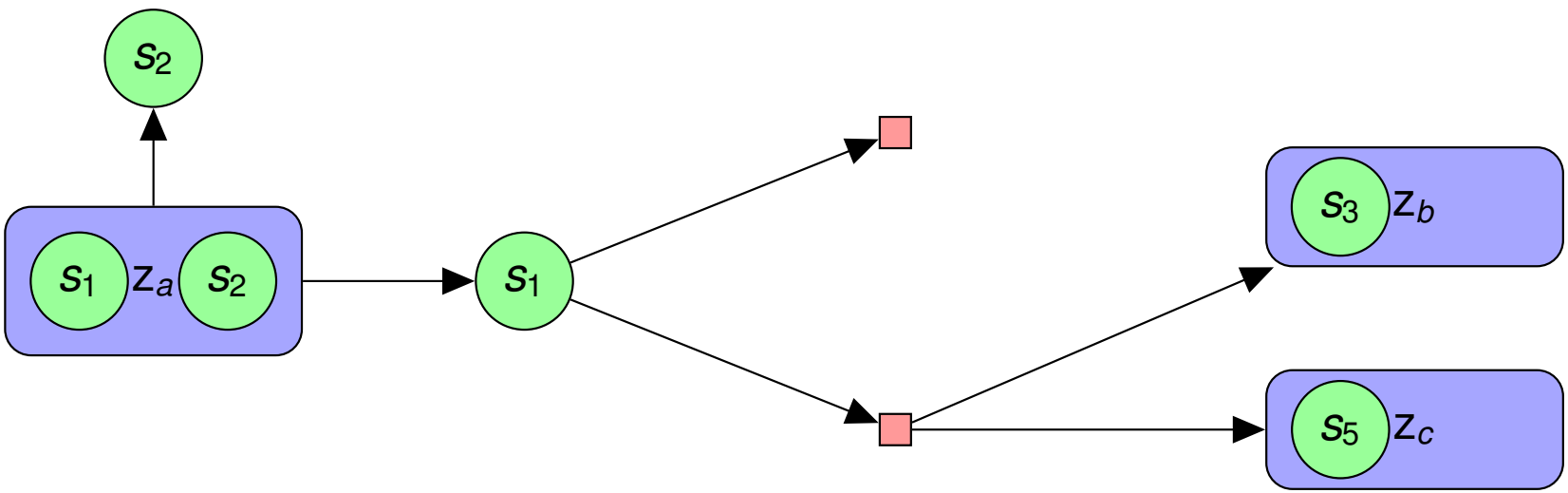

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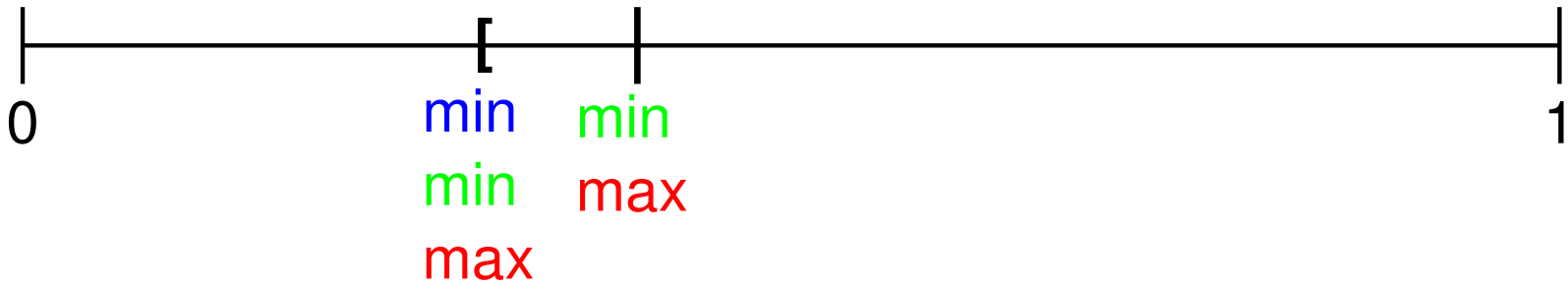

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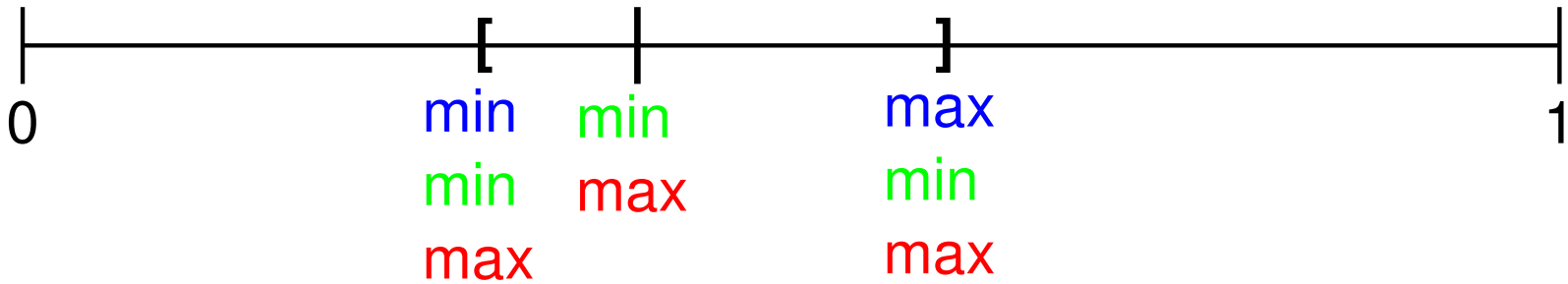

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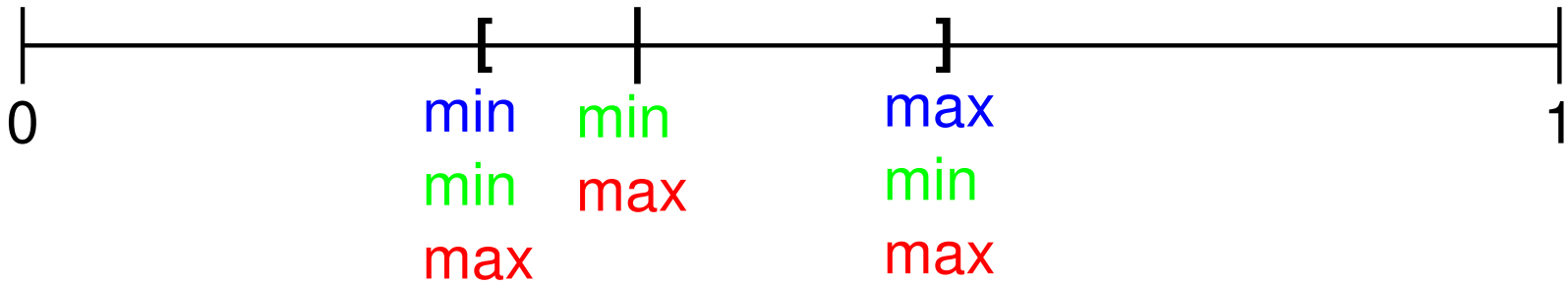
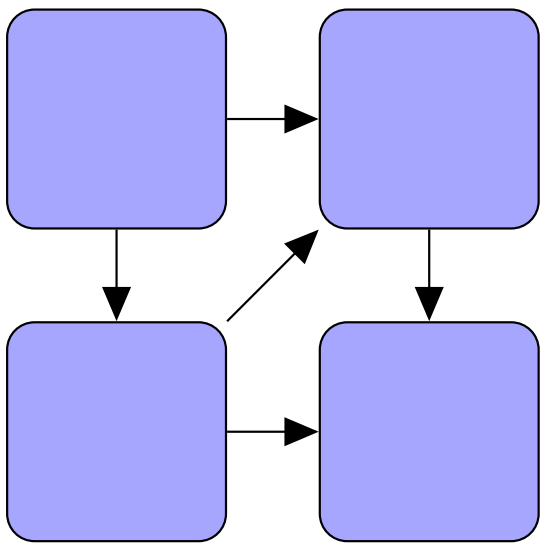


- $\Rightarrow$ finite probabilistic two-player games
- efficient solution techniques exist
  (value iteration or policy iteration)

GBA: Refinement

- bounds may be too coarse



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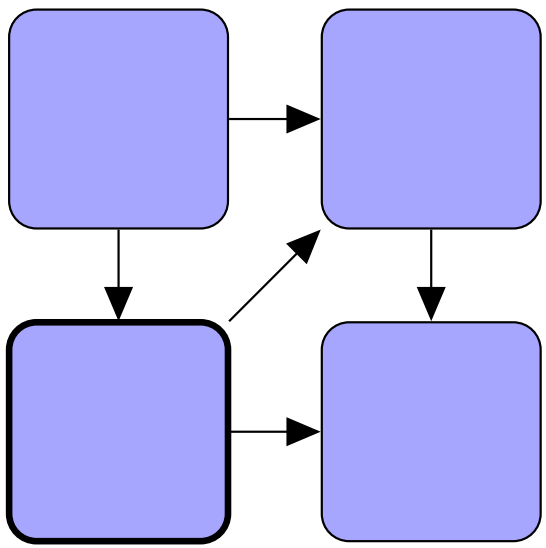

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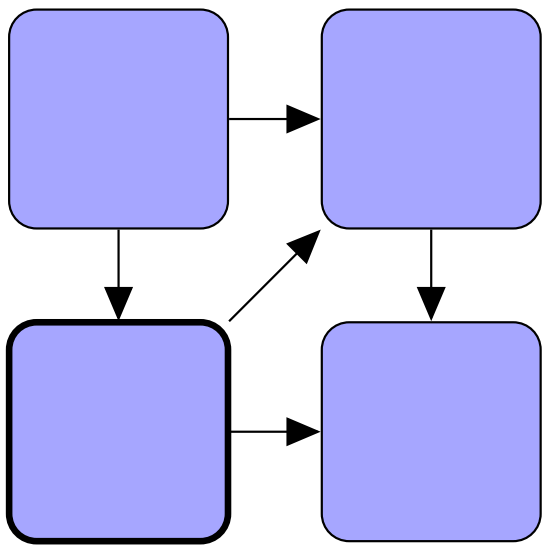

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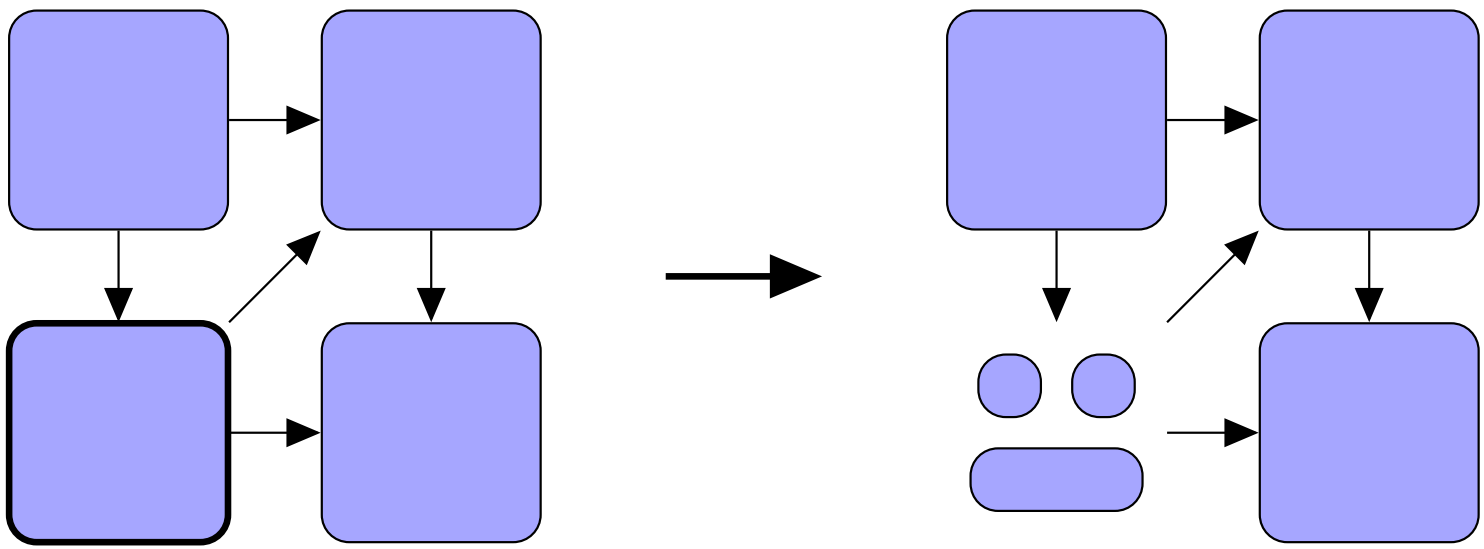

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- choice depends on probabilities and strategies
- split into new states
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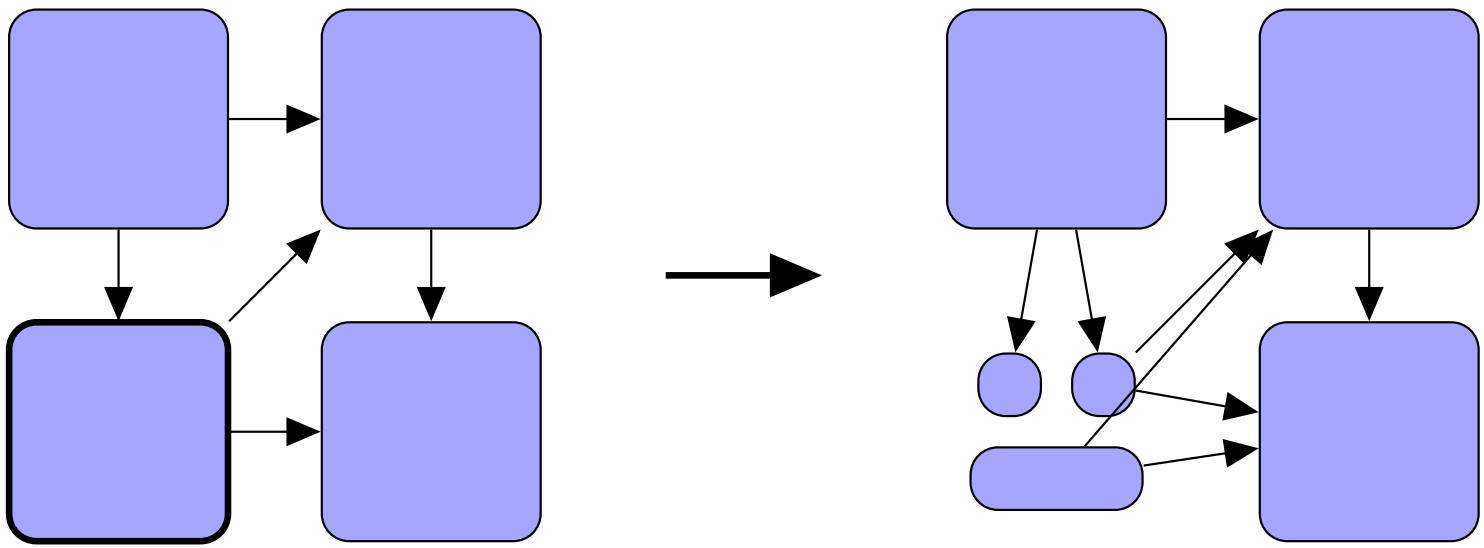

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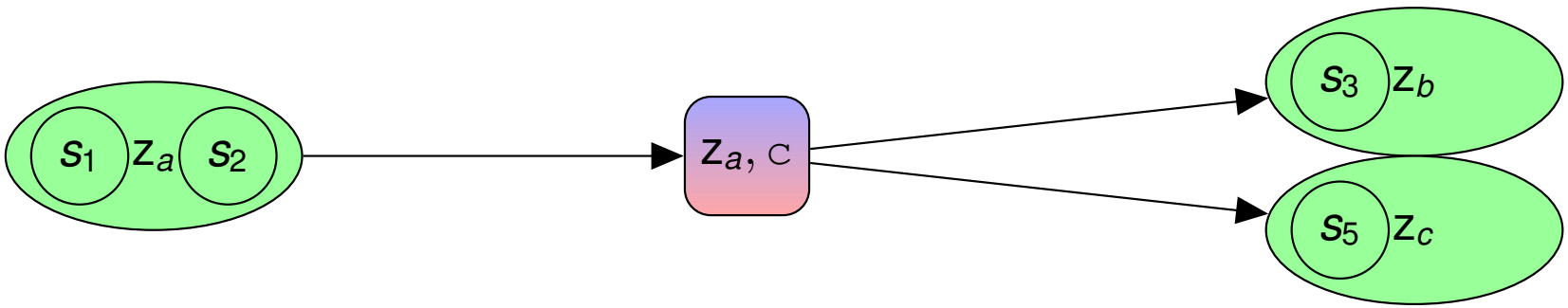

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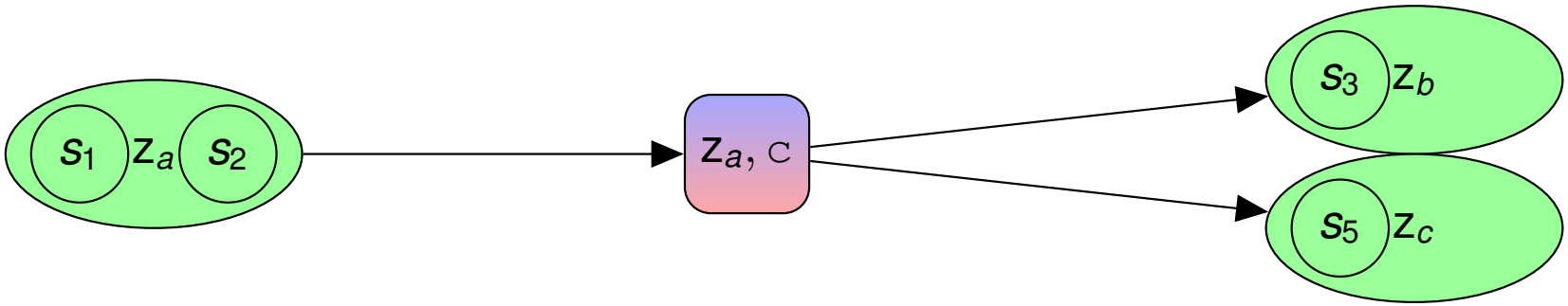

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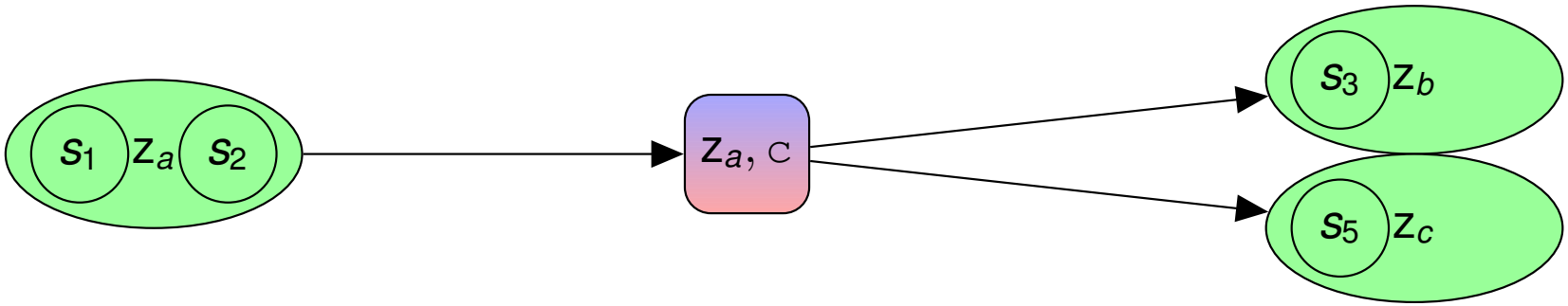

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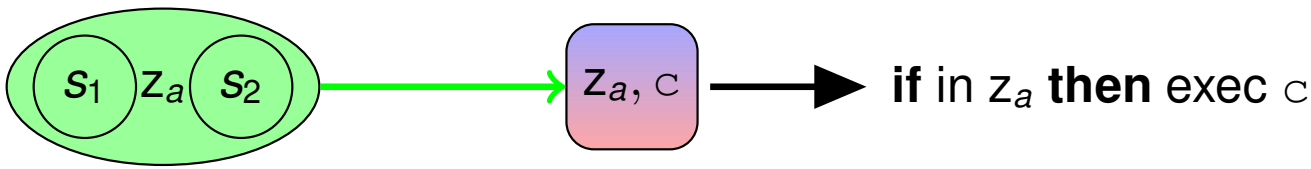
Extensions

- controller synthesis



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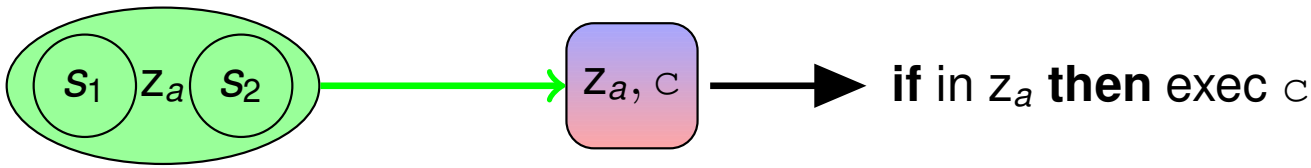

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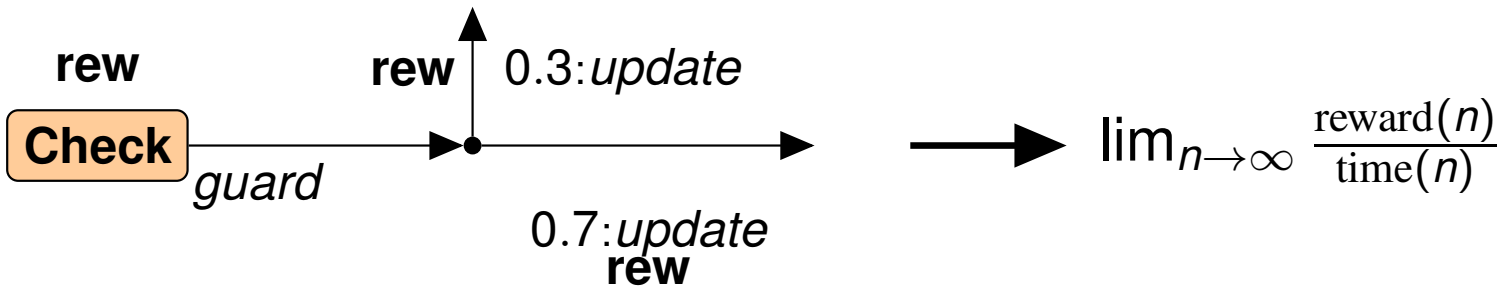

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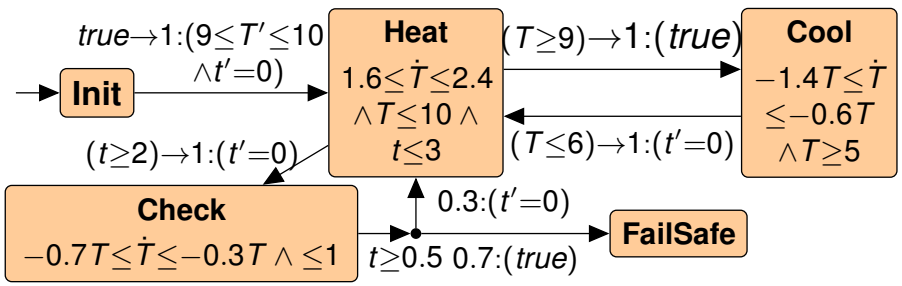

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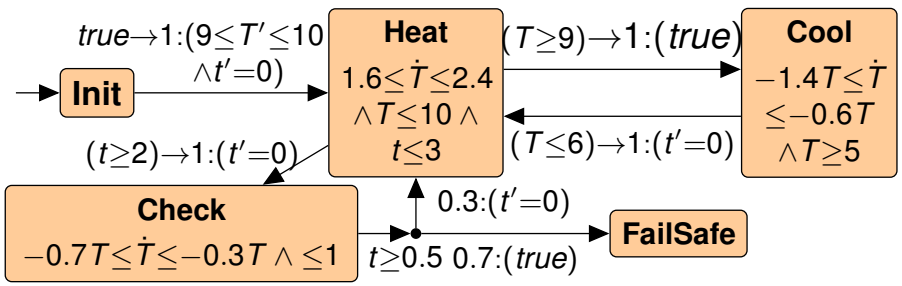
- maximal probability reach safe state within given time B



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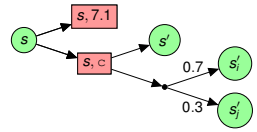
coarseness of abstraction: PHAVer-specific

Summary

- game semantics for probabilistic hybrid systems



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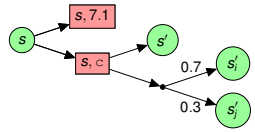

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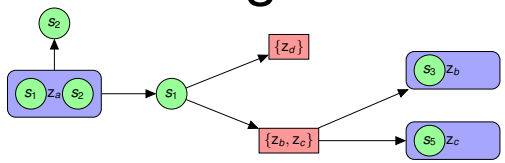

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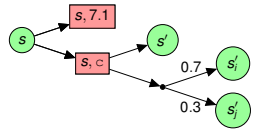

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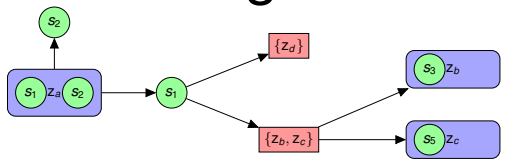

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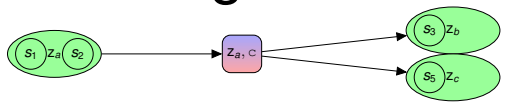

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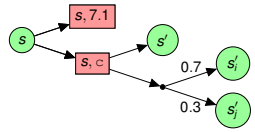

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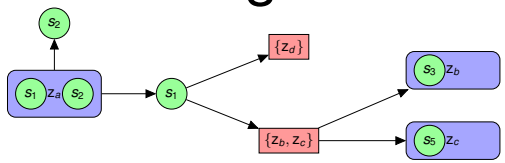

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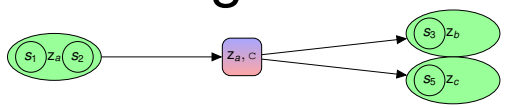

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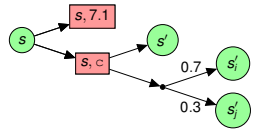

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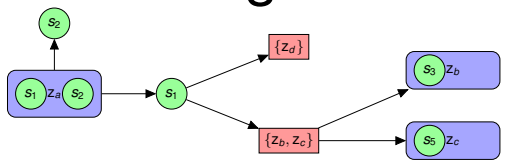

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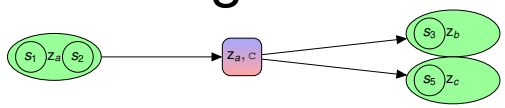

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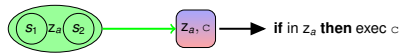

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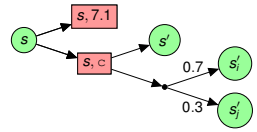

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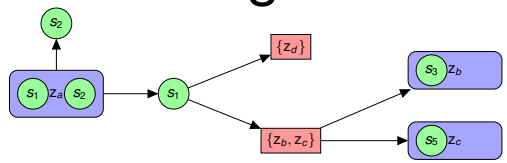

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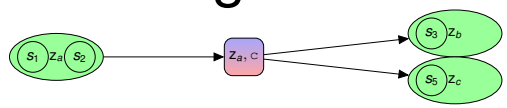

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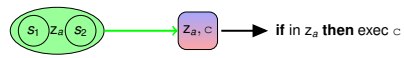

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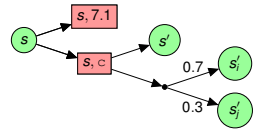

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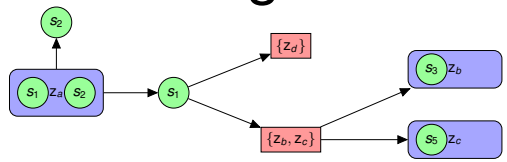

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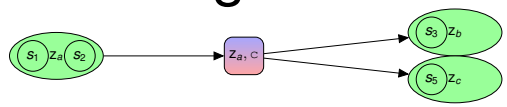

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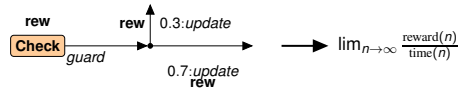

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