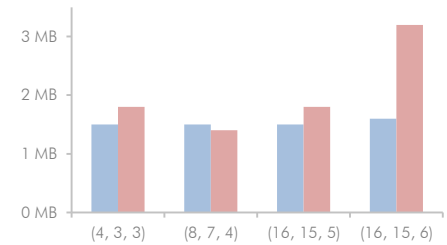
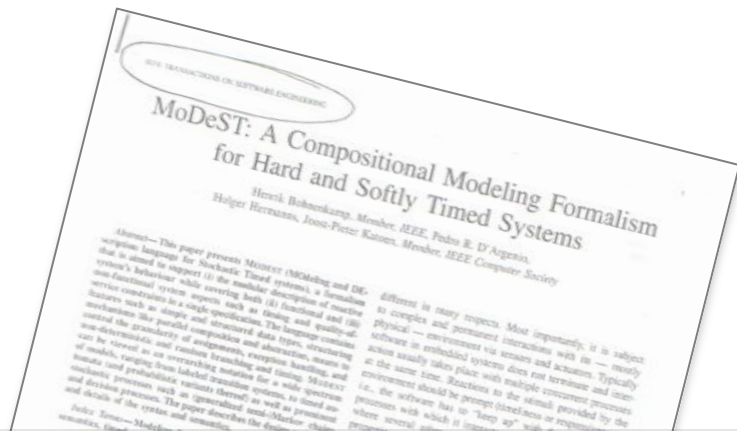


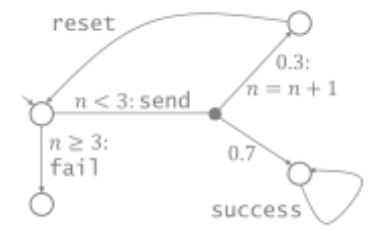
# Partial Order Methods for Statistical Model Checking and Simulation



1

The model

The analysis techniques



2

3

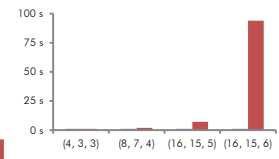
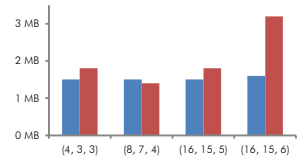


Idea + Solution

4

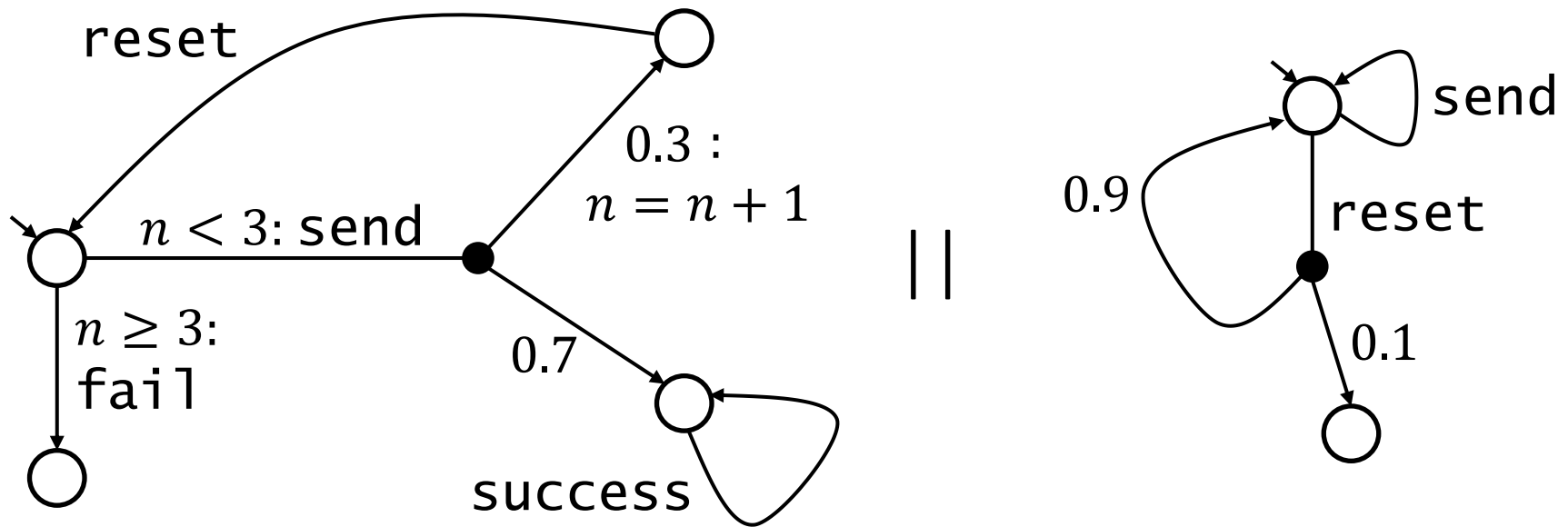
Evaluation

2 case studies



# The Model

Networks of probabilistic automata with variables <sup>PA/MDP</sup>



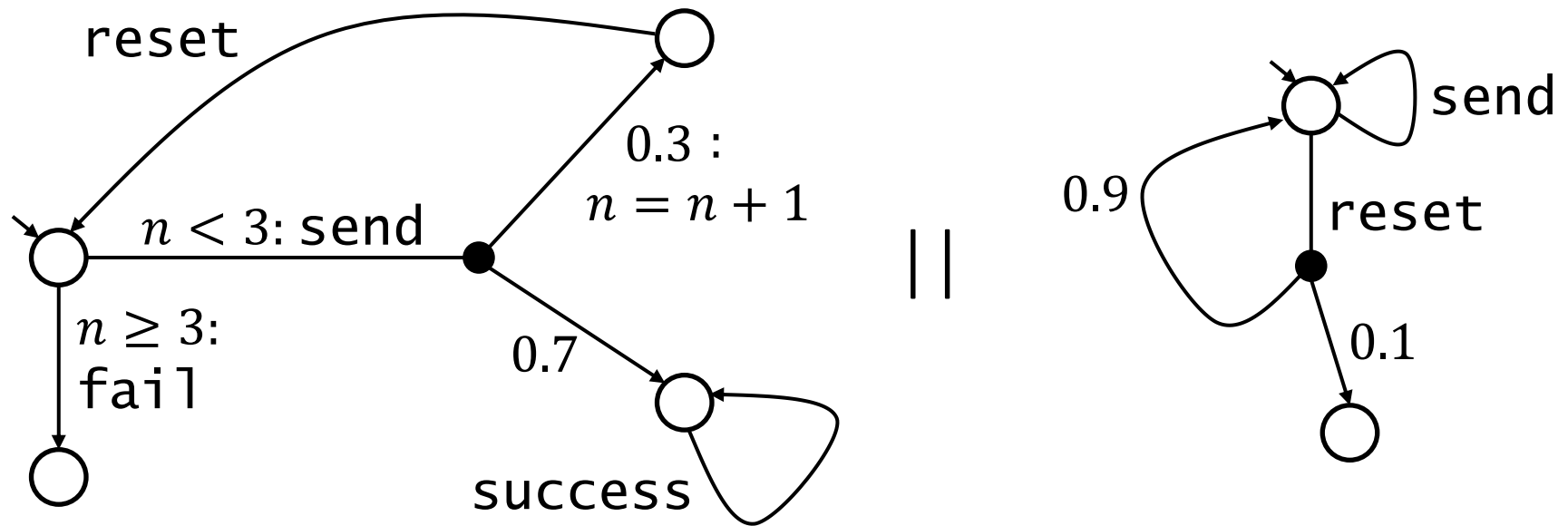
$P_{\leq 10}(\text{success}) > 0.9 ?$        $P_{\leq 10}(\text{deadlock}) \leq 0.05 ?$

Synchronisation, discrete variables (possibly shared)

Transitions lead to distributions:  $\rightarrow \subseteq S \times \Sigma \times \text{Dist}(S)$

# The Analysis Techniques

Simulation: exploring concrete paths through the model



$P_{\leq 10}(\text{success}) > 0.9 ?$

yes yes yes no  $\Rightarrow$  **no**?

$P_{\leq 10}(\text{deadlock}) \leq 0.05 ?$


how many runs needed?

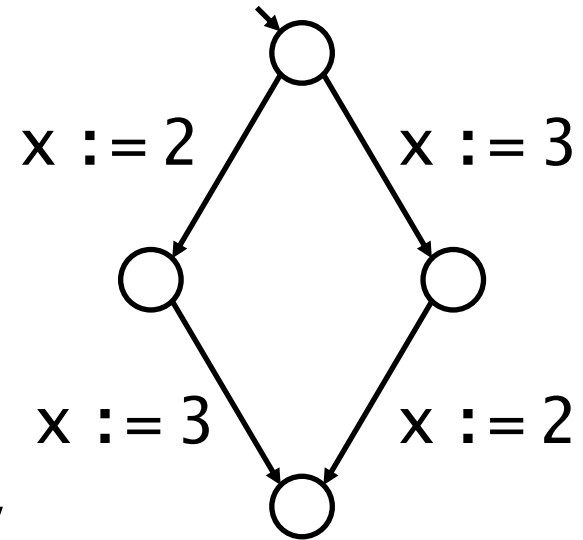
+ advanced statistics: Statistical Model Checking **SMC**

# The Problem

PA can be nondeterministic

⇒ "paths": trees?

  
simulation



Standard solution: simulate anyway

⇒ implicit *scheduler* resolves nondeterminism **bad idea**

$$P(x \geq 3)$$

$$P(x \geq 3) = 0$$

$$P(x \geq 3) = 1$$

...

???

more motivation and examples: see H. (FMCO 2010)

Partial Order Methods for Statistical Mo

Model-Checking and Simulation  
for Stochastic Timed Systems

Axel Hartmann

Saarland University - Computer Science, Saarbrücken, Germany

Abstract. For verification and performance evaluation, system models that can express stochastic as well as real-time behaviour are of increasing importance. Although an integrated stochastic-timed verification procedure is highly desirable, both model-checking and simulation of providing a complete, fully automatic verification procedure for the use of

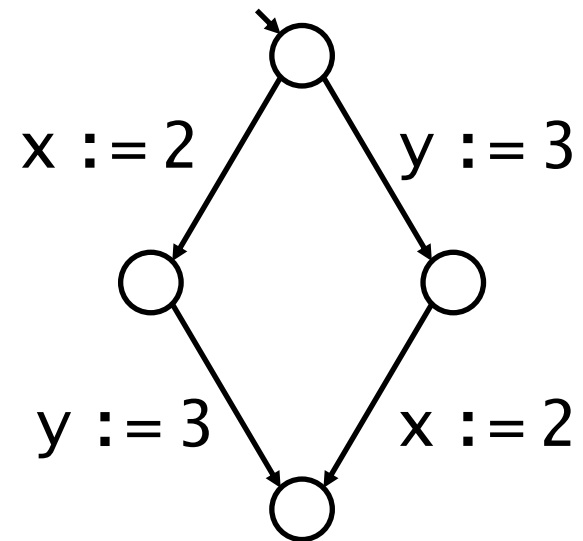
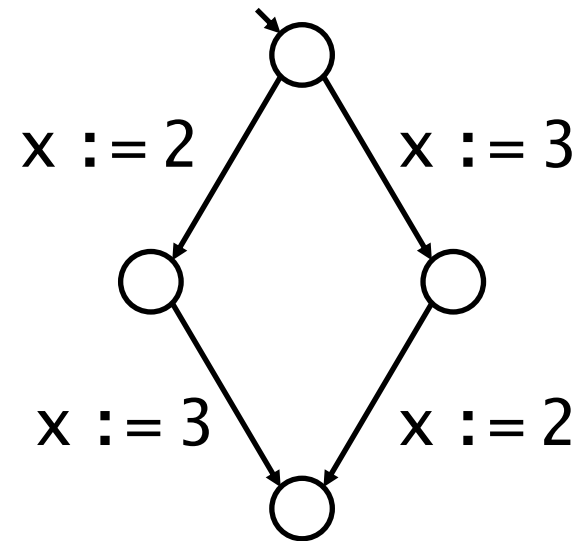
# The Idea

Nondeterminism may be spurious  
= irrelevant for the results



try to detect spuriousness  
on-the-fly and ignore  
using methods derived  
from partial order reduction

$$P(x \geq 3) = 0 \quad \blacktriangleleft$$



# The Idea

## Partial order reduction<sup>1</sup>

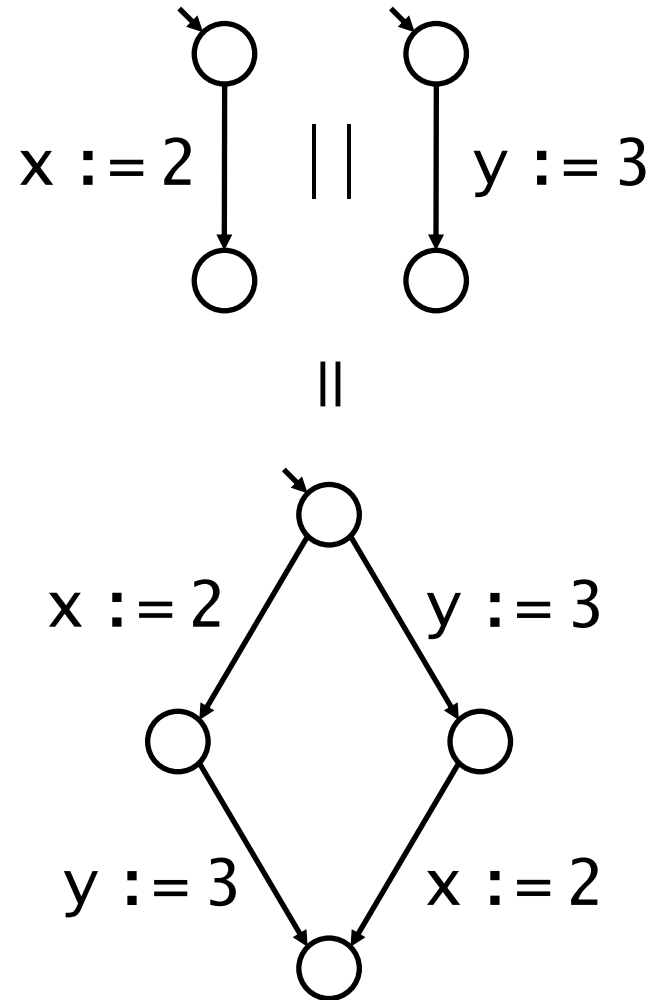
Speed up model checking by removing spurious paths

Implemented on-the-fly during state-space generation *e.g. in SPIN*

**This looks precisely like what we want!**

Limitation:

In practice, can only deal with spurious interleavings



<sup>1</sup> Developed concurrently by Godefroid, Peled and Valmari

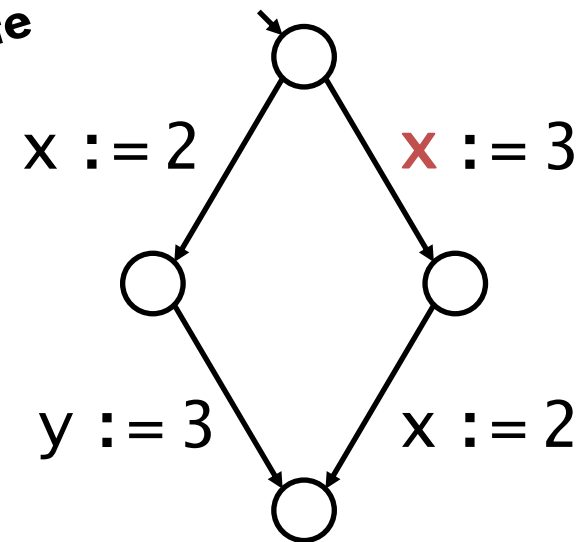
## Detecting spuriousness **on-the-fly** via ample sets<sup>1</sup>

A1 If  $\text{ample}(s) \neq T(s)$ , then all its transitions are invisible

A2 On all original paths, transitions in  $\text{ample}(s)$  must occur before transitions *dependent* on  $\text{ample}(s)$  **need to bound lookahead**

A3 Every end component has a state  $s$  such that  $\text{ample}(s) = T(s)$  **over-approximate**

A4 If  $\text{ample}(s) \neq T(s)$ , then  $|\text{ample}(s)| = 1 = \mathbf{determinism!}$

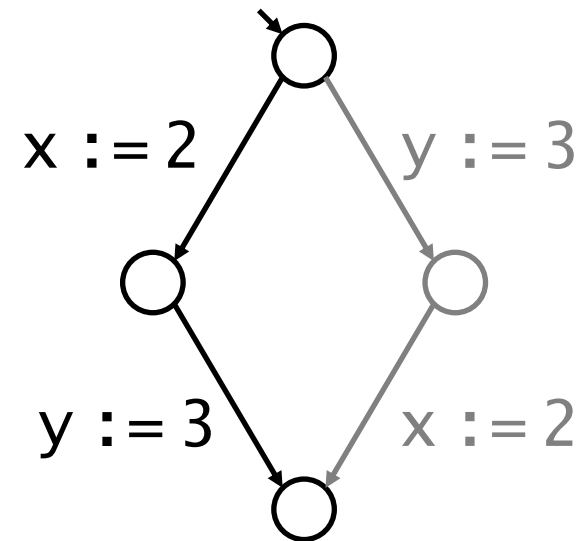


<sup>1</sup> D. Peled (CAV 1994) / Baier, D'Argenio, Größer (ENTCS 2006)



## Detecting spuriousness on-the-fly via ample sets

- A1 If  $\text{ample}(s) \neq T(s)$ , then all its transitions are invisible
- A2' All original paths have a prefix of length  $\leq k$  on which a transition in  $\text{ample}(s)$  occurs before transitions...
- A3' Of the last  $l$  states, there is at least one with  $\text{ample}(s) = T(s)$
- A4' If  $\text{ample}(s) \neq T(s)$ , then  $|\text{ample}(s)| = 1$  or  $T(s) = \emptyset$
- ⇒ Two parameters:  $k$  and  $l$   
 $k_{min}$ : minimal  $k$  necessary

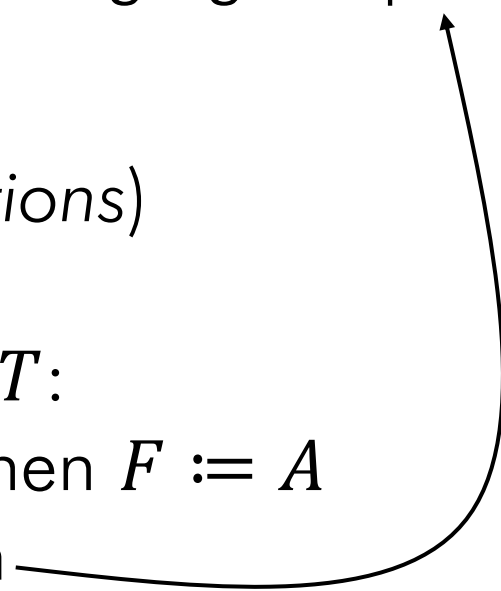


## On-the-fly partial order simulation algorithm

User need not worry about  $k$  and  $l$ :

```
 $s :=$  initial state
do
   $F :=$  null
   $T := Succ(s)$  (succ. distributions)
  if  $|T| = 1$ , then  $F := T$ 
  else for each singleton  $A \subseteq T$ :
    if  $s$  is a valid ample set, then  $F := A$ 
  if  $F =$  null, abort simulation
  else  $s := sample(F)$ 
until termination criterion satisfied
```

On abort because  $k$  or  $l$  exceeded,  
error message gives precise reason



# The Solution

## Implementation

Implemented in modes,

the discrete-event simulator for the Modest<sup>1</sup> language



[www.modestchecker.net](http://www.modestchecker.net)

Part of the Modest toolset

- modes: Simulation
- mcpta: Model checking
- mime: Graphical interface



<sup>1</sup> Bohnenkamp, D'Argenio, Hermanns, Katoen (IEEE TSE 2006)

# The Solution

## Case studies

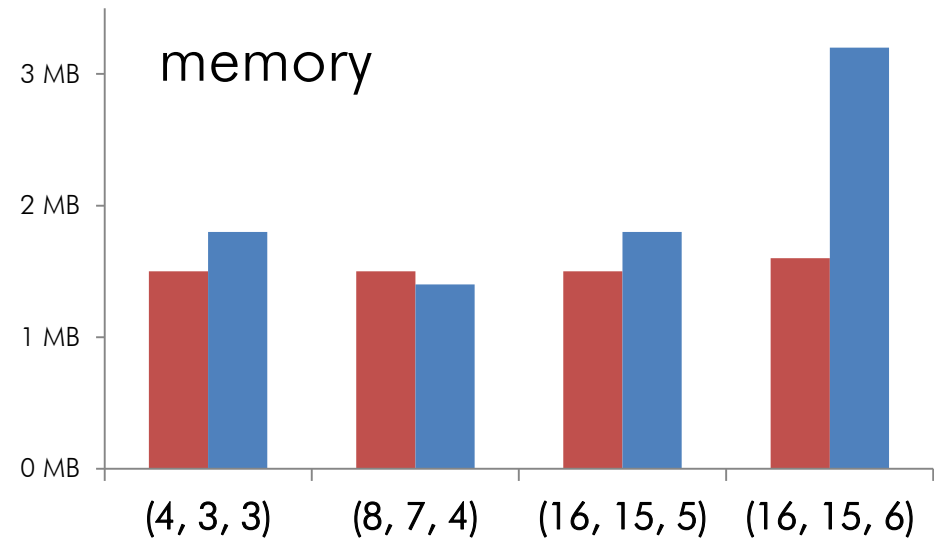
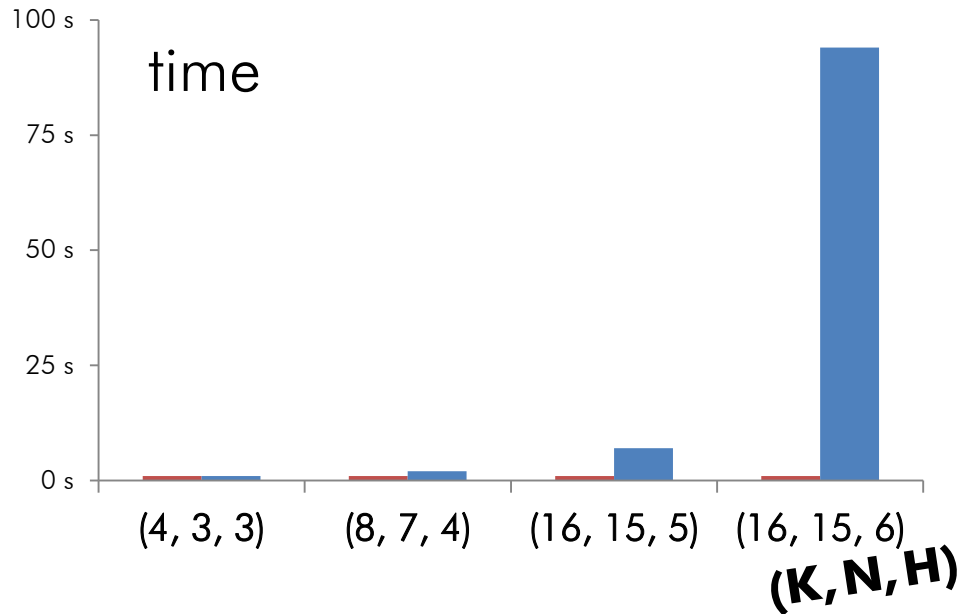
IEEE 802.3 BEB protocol

**implicit-scheduler** vs.  
**partial-order** simulation

$k_{min} = H$  (number of hosts)

**prototype  
implementation**

*Model checking: infeasible  
> 100 GB for (16, 15, 5)*



## Case studies

ARCADE<sup>1</sup> dependability evaluation models

Stochastic timed systems, lots of component automata

↳ *orthogonal to our method*

model		†	implicit-sched.	partial-order	$l$	$k_{min}$
1bc-1rudded	3	100	1 s / 1.3MB	1 s / 1.5MB	16	2
2bc-1rufcfs	4	100	2 s / 1.1MB	4 s / 1.1MB	16	3
dda-scaled	41	15	41 s / 1.5MB	379 s / 2.6MB	16	6
rsc-scaled	36	15	24 s / 1.6MB	132 s / 2.0MB	16	4

<sup>1</sup> Boudali, Crouzen, Haverkort, Kuntz, Stoëlinga (DSN 2008)

# The Conclusion

Simulation for nondeterministic models<sup>PA</sup>

Previously: Impossible – or wrong results **implicit scheduler**

Now: Possible – for spurious interleavings  **partial order reduction**

– Time and memory penalty

+ Applies to any simulation-based approach **statistical model checking**

*“A little bit of on-the-fly model checking during simulation”*

Implementation (prototype) available at

[www.modestchecker.net](http://www.modestchecker.net)