The Symbolic Analysis of Hybrid Automata

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Two Parallel Stories

Symbolic model checking:

-abstract data type *region algebra* -termination analysis

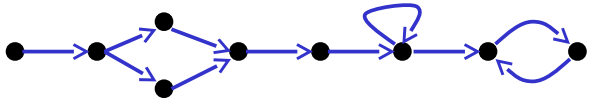
Theory

Timed and hybrid systems:

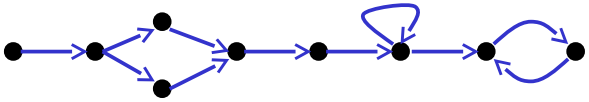
specific region algebra (e.g. clock regions, polyhedra)

Application

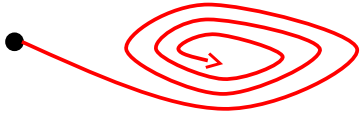
Discrete (transition) system



Discrete (transition) system

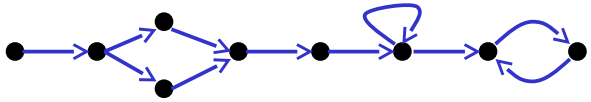


Continuous (dynamical) system



$Q = R^n$

Discrete (transition) system



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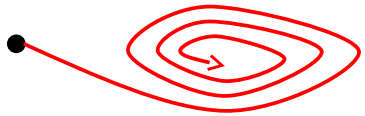

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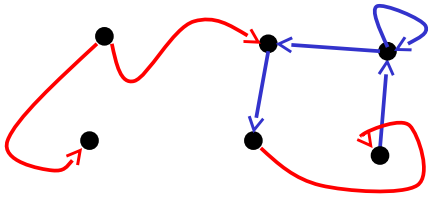
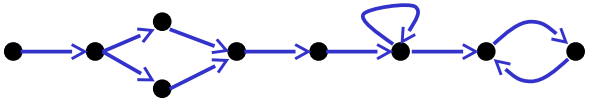
Hybrid system



jumps
flows

Discrete (transition) system



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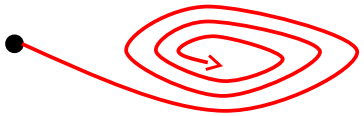

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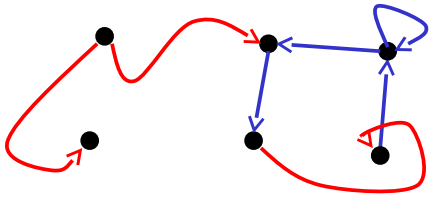
Hybrid system



jumps
flows

-
nondeterministi
-time abstract

A Thermostat

States

$x \in R$	temperature
$h \in \{ on, off \}$	heat

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Flows

f_1	[] h = on	$\rightarrow$ x' = K·(H-x)
f_2	[] h = off	$\rightarrow$ x' = -K·x

invariant
s

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$x \in R$	temperature
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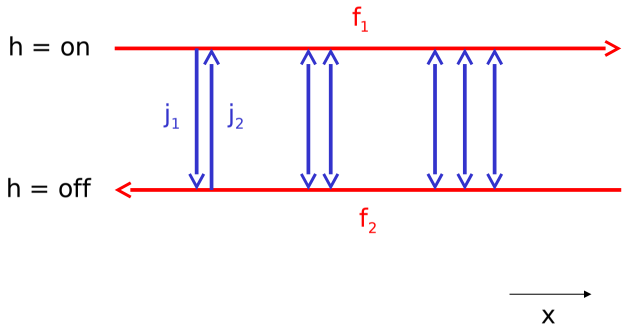
Jumps

j_1	[] h = on	→	h := off
j_2	[] h = off	→	h := on

guards



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$h \in \{ on, off \}$	heat
$t \in R$	timer (“clock”)

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Jumps

j_1	[] h = on	→	h := off
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$x \in R$	temperature
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Flows

f_1	[] h = on	→	x' = K·(H-x); t' = 1
f_2	[] h = off	→	x' = -K·x; t' = 1

Jumps

j_1	[] h = on ∧ t ≥ L	→	h := off; t := 0
j_2	[] h = off ∧ t ≥ L	→	h := on; t := 0

A Thermostat

States

$x \in R$	temperature
$h \in \{ on, off \}$	heat
$t \in R$	timer (“clock”)

Flows

f_1 [] $h = on \wedge t \le U \rightarrow x' = K \cdot (H - x);\ t' = 1$

f_2 [] $h = off \wedge t \le U \rightarrow x' = -K \cdot x;\ t' = 1$

Jumps

j_1 [] $h = on \wedge t \ge L \rightarrow h := off;\ t := 0$

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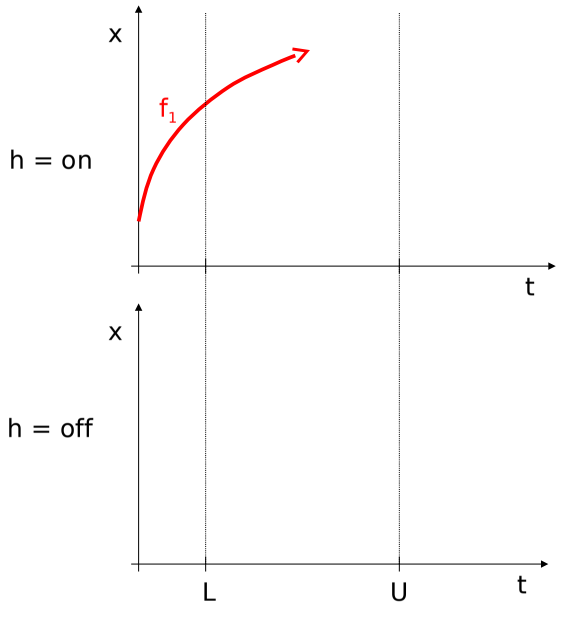
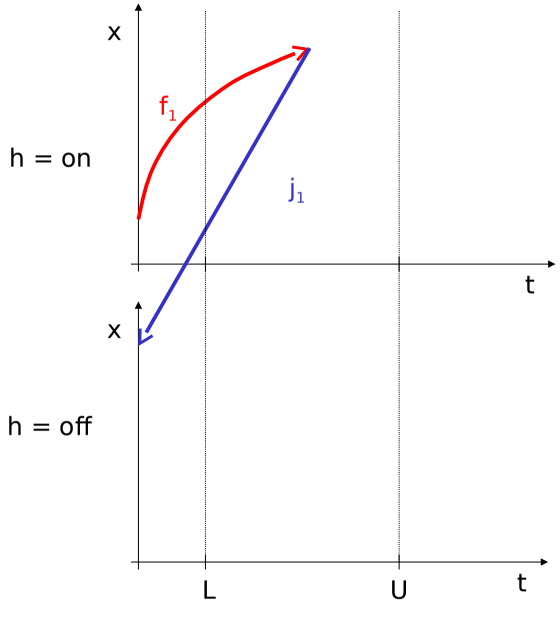
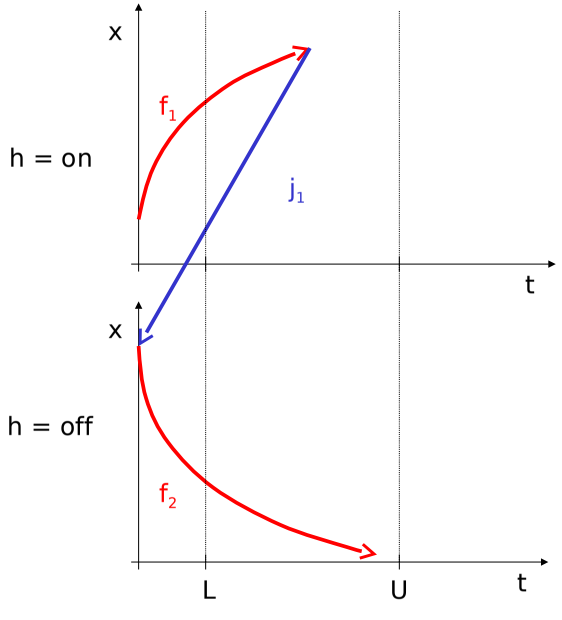
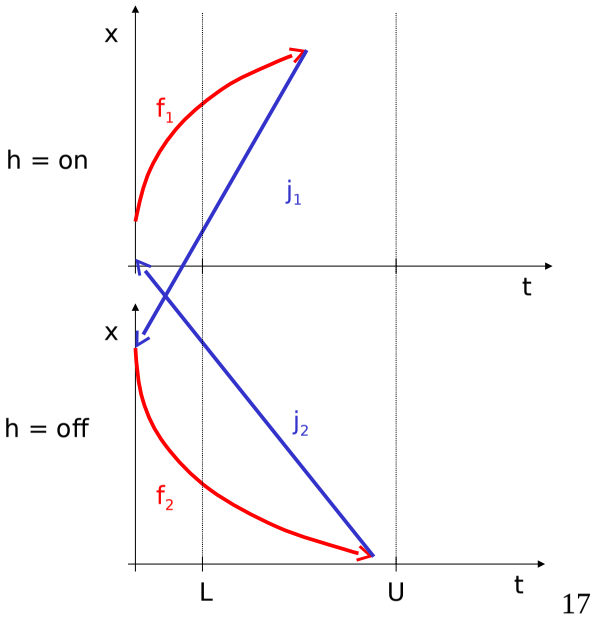
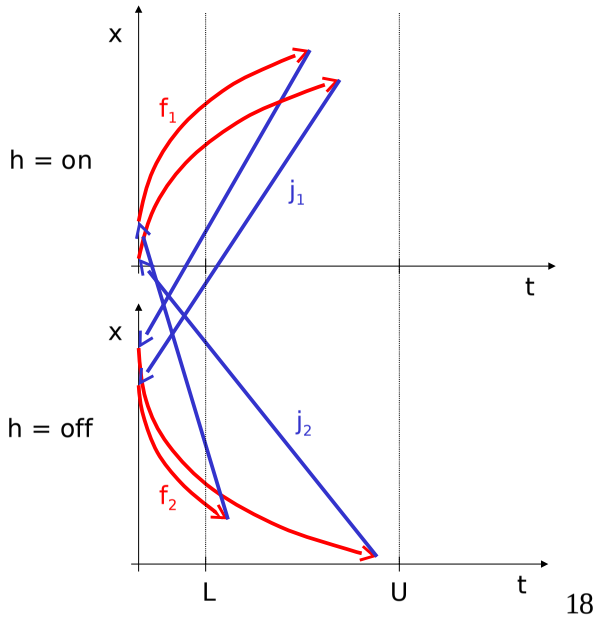




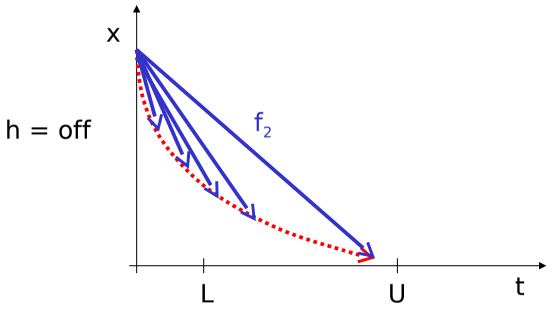






From a Hybrid System to a Symbolic Transition System

1. Discretize: from continuous to discrete 2. Lift: from states to state sets (“regions”) 3. Observe: from infinite to finititary



Step 1: Discretize

Transition System

Q	set of states
Σ	set of actions
post: $Q \times \Sigma \to 2^Q$ function	successor

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$Q = R^2 \times \{ on, off \}$

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$$\text{post}(x, t, on, j_1) = \begin{cases} \{ (x, 0, off) \} & \text{if } t \geq L \\ \varnothing & \text{if } t < L \end{cases}$$

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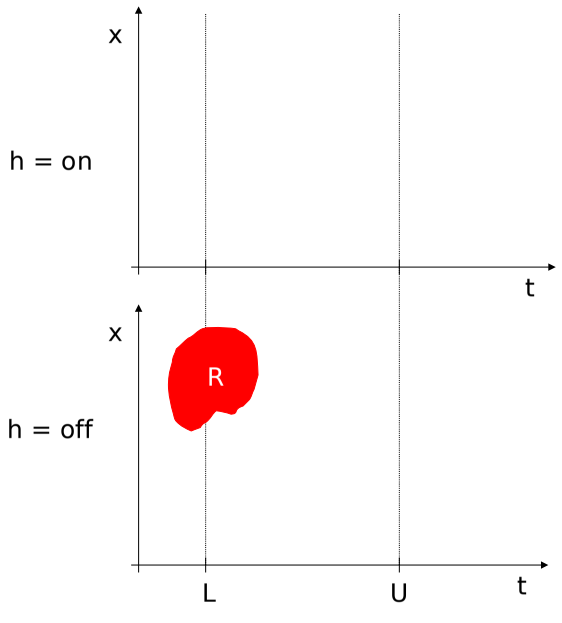
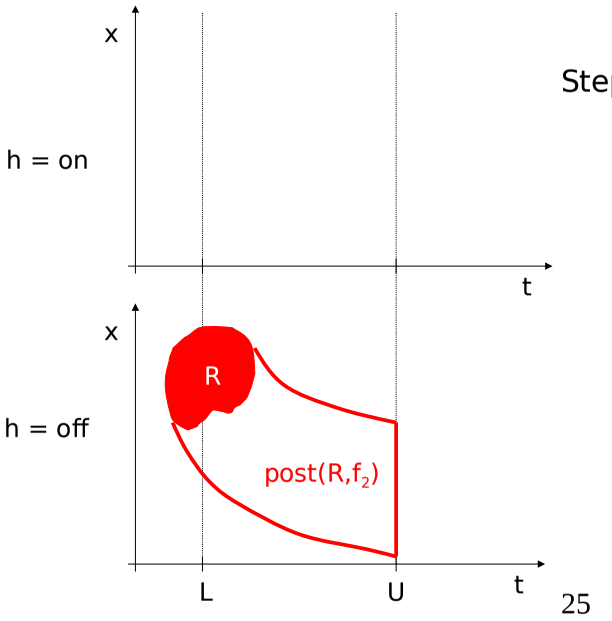
$$\text{post}(x, t, on, f_1) = \begin{cases} \text{infinite set} & \text{if } t < U \\ \{ (x, t, on) \} & \text{if } t = U \\ \varnothing & \text{if } t > U \end{cases}$$



Step 2: Lift



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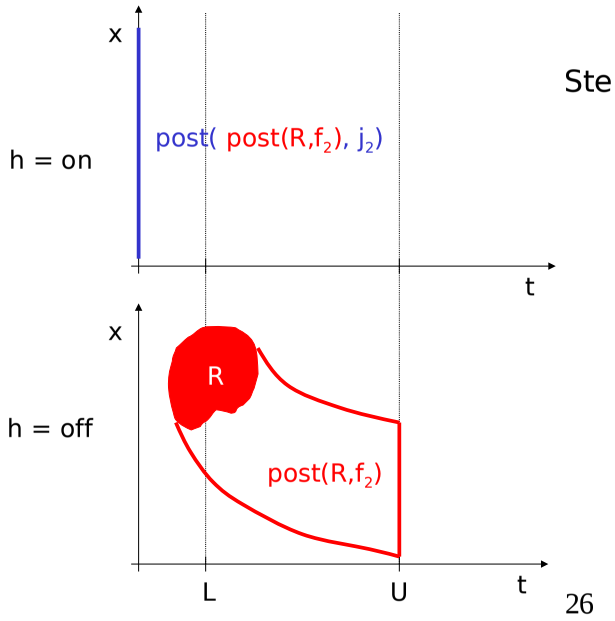

Step 2: Lift

Lifted Transition System

Q

Σ

post: $2^Q \times \Sigma \to 2^Q$ $\quad post(R,\sigma) = \cup_{q \in R}\, post(q,\sigma)$

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Σ

post: $2^Q \times \Sigma \to 2^Q$ $\quad post(R,\sigma) = \cup_{q \in R}\, post(q,\sigma)$

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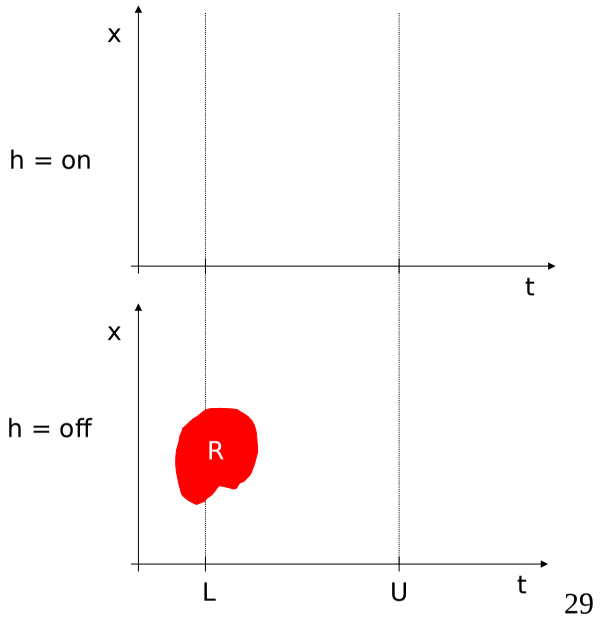
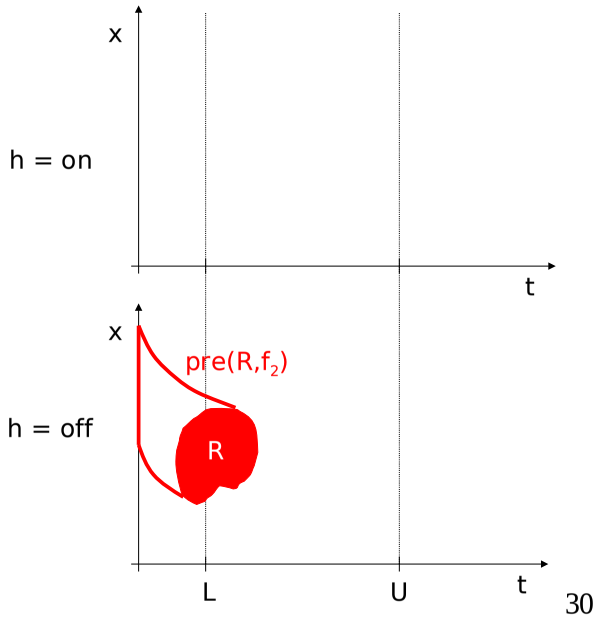
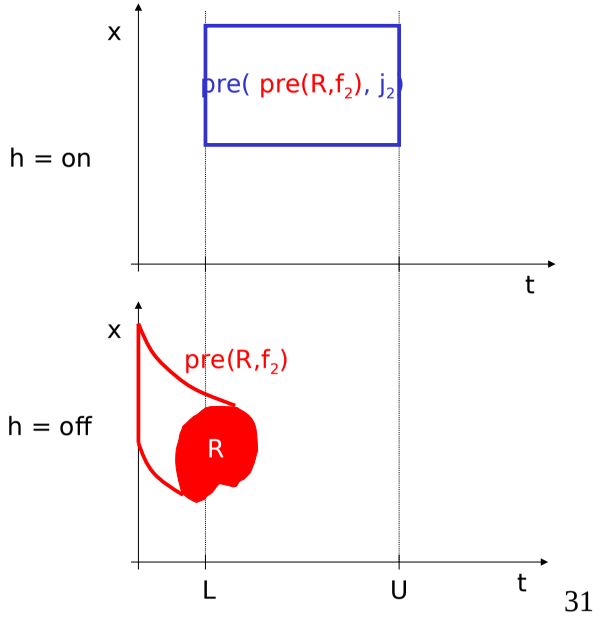

x
h = on
t
x
h = off
R
L
U
t

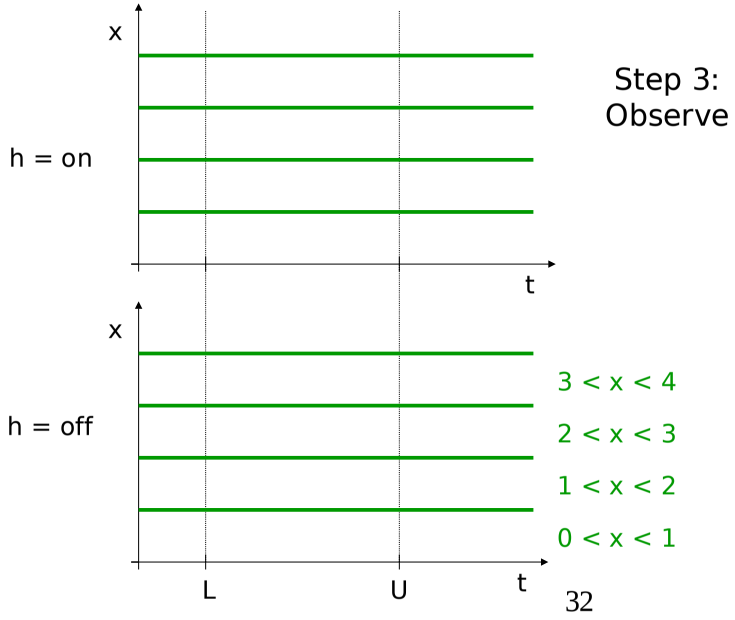
29


x
h = on
t
x
pre(R,f2)
h = off
R
L
U
t

30


x
pre(pre(R,f2), j2)
h = on
t
x
pre(R,f2)
h = off
R
L
U
t

31


x
h = on
t
x
h = off
t
L
U
t
Step 3:
Observe
3 < x < 4
2 < x < 3
1 < x < 2
0 < x < 1

32

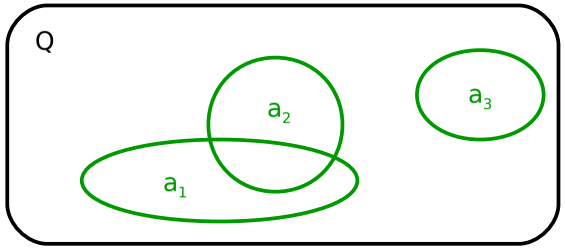
Observed Transition System

Q

Σ

pre, post: $2^Q \times \Sigma \rightarrow 2^Q$

A = { a_1, a_2, a_3, ... } set of observations $a_i \mu Q$



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Σ

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Thermostat

A = { on, off } [{ x = c, c < x < c+1 | c ∈ Z }

Model Checking: From Finite-state to Hybrid Systems

Graph Algorithms:

-unit operation: access to a vertex (“state”) or edge (“transition”) -for finite-state systems

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Graph Algorithms:

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Symbolic Algorithms:

-unit operation: pre or post on a state set ("region")
-also for infinite-state systems
-two ingredients:

1. region algebra (e.g. BDDs, clock zones, polyhedra) 2. termination analysis

Symbolic Transition System

Q

Σ

pre, post

A

$\Re = \{ R_1, R_2, \ldots \}$

set of regions $R_i \subseteq Q$

Symbolic Transition System

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Σ

pre, post

A

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Region algebra:

1. $A \subseteq \Re$
2. pre, post: $\Re \times \Sigma \to \Re$ computable
3. $\wedge : \Re^2 \to \Re$
   $\setminus : \Re^2 \to \Re$
   computable
   $\subseteq : \Re^2 \to \{ t, f \}$

Symbolic Transition System

1. Local computation: Region Operations

Compute pre, post, $\wedge$, $\setminus$, and $\subseteq$ on regions in $\Re$.

Symbolic Transition System

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Compute pre, post, Å , \ , and $\subseteq$ on regions in $\Re$.

2. Global computation: Symbolic Semi-Algorithms

Starting from the observations in A, compute new regions in $\Re$ by applying the operations pre, post, Å , \ , and $\subseteq$.

Region Algebras

If -Q is the valuations for a set X:Vals of typed variables,
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This is because each pre and post operation is a quantifier elimination:

$$pre(R(X)) = (\exists \underline{X})\ (Trans(X,\underline{X}) \wedge R(\underline{X}))$$

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Example: boolean systems
(Vals = B, and $\Re$ = boolean expressions over X)

Example: Polyhedral Hybrid Automata

$Q = B^m \times R^n$

Invariants and guards: boolean and linear constraints, e.g. $a \wedge (3x_1 + x_2 \leq 7)$

Flows: rectangular differential inclusions, e.g. $x'_1 \in [1,2]$ Jumps: boolean and linear constraints, e.g. $x_2 := 2x_1 + x_2 + 1$

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$= (\exists \underline{x}_1, \underline{x}_2)\ (x_1 \le x_2 \wedge \underline{x}_1 = x_1 \wedge \underline{x}_2 = 2x_1\text{-}1 \wedge 1 \le \underline{x}_1 \le \underline{x}_2 \le 2)$

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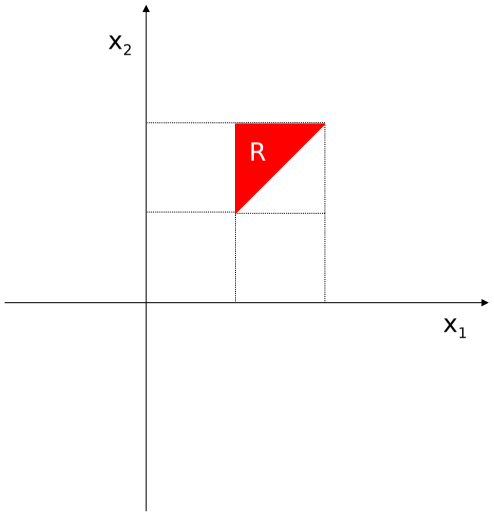
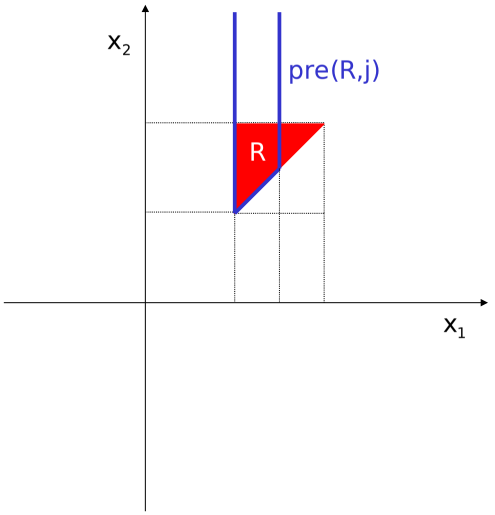
$= x_1 \le x_2 \wedge 1 \le x_1 \le 3/2$





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pre($1 \le x_1 \le x_2 \le 2$, f)

$= (\exists\ 1 \le k_1 \le 2)\ (\exists\ \delta \ge 0)\ (1 \le x_1 + k_1\delta \le x_2 + \delta \le 2 \wedge (\forall\ 0 \le \varepsilon \le \delta)\ (x_1 + k_1\varepsilon \le x_2 + \varepsilon))$

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$= (\exists\ \delta \ge 0)\ (\exists\ \delta \le d_1 \le 2\delta)\ (1 \le x_1 + d_1 \le x_2 + \delta \le 2 \wedge$

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$= (\exists\ \delta \ge 0)\ (\exists\ \delta \le d_1 \le 2\delta)\ (1 \le x_1 + d_1 \le x_2 + \delta \le 2 \wedge x_1 \le x_2 \wedge x_1 + d_1 \le x_2 + \delta)$

convex invariants

Example: Polyhedral Hybrid Automata

Flow f: $\square\, x_1 \le x_2 \rightarrow x'_1 \in [1,2];\ x'_2 = 1$

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$= (\exists\ \delta)\ (9\ d_1)\ (0 \cdot \delta \cdot d_1 \cdot 2\delta\ \text{Æ}\ 1 \le x_1 + d_1 \le x_2 + \delta \le 2 \wedge x_1 \le x_2)$

convex guards

Example: Polyhedral Hybrid Automata

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$= (9\ \delta)\ (0 \cdot \delta\ \text{Æ}\ 1 \cdot x_2 + \delta \cdot 2\ \text{Æ}\ x_1 \cdot x_2\ \text{Æ}\ \{\delta, 1 - x_1\} \cdot \{2\delta, x_2 - x_1 + \delta\})$

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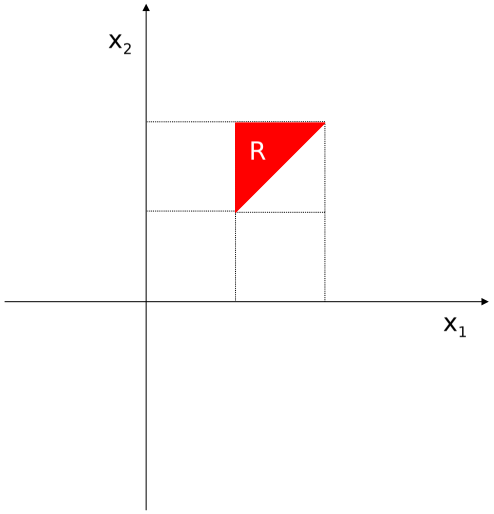

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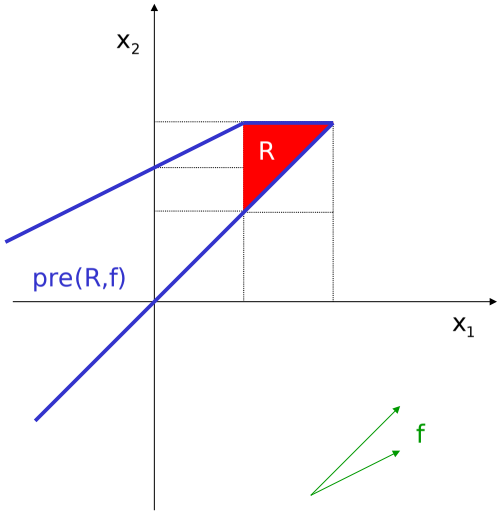
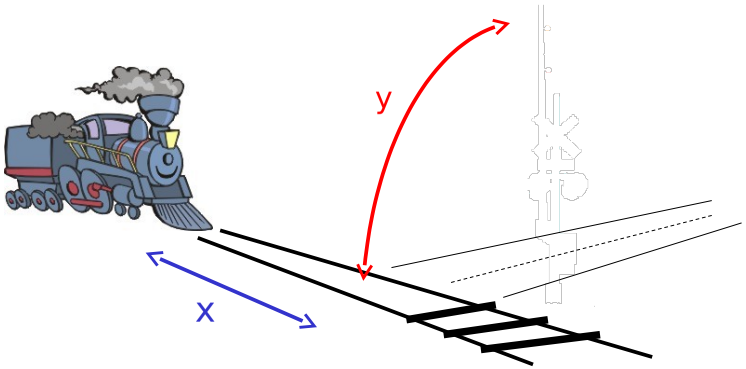
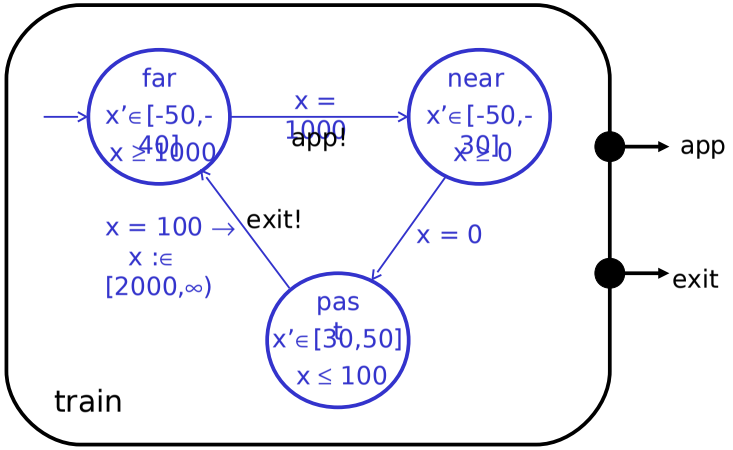




69



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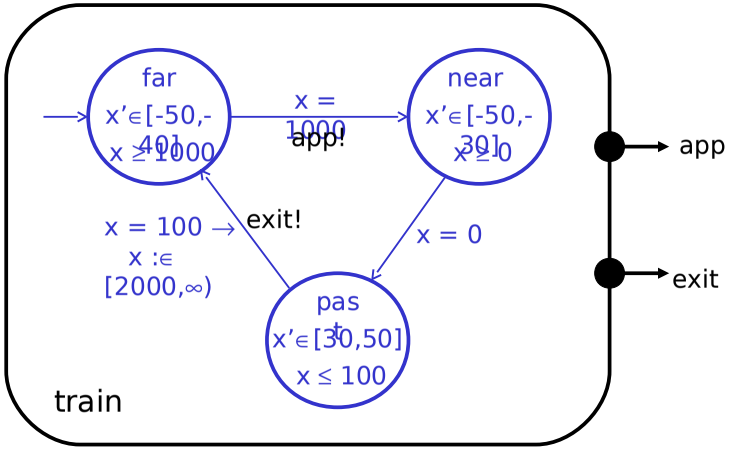
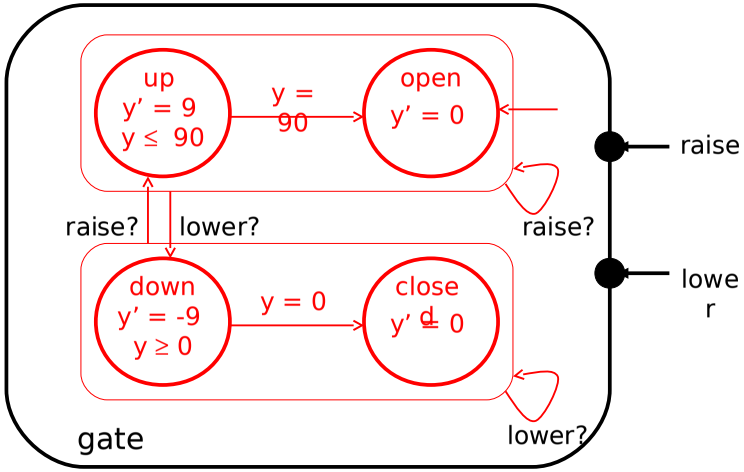


x: initialized rectangular variable

71



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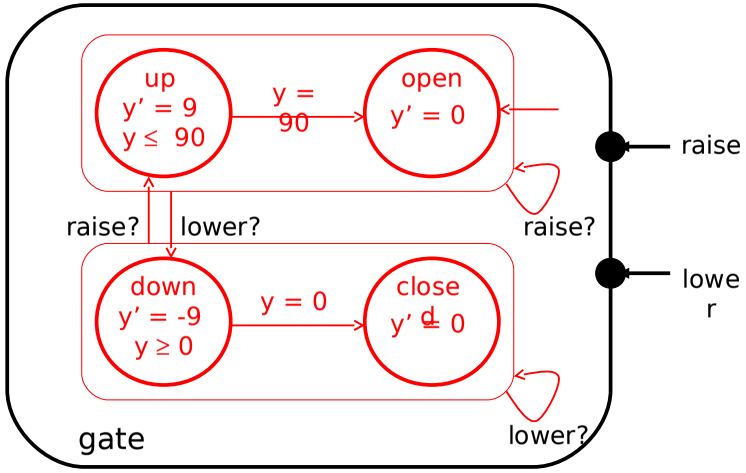
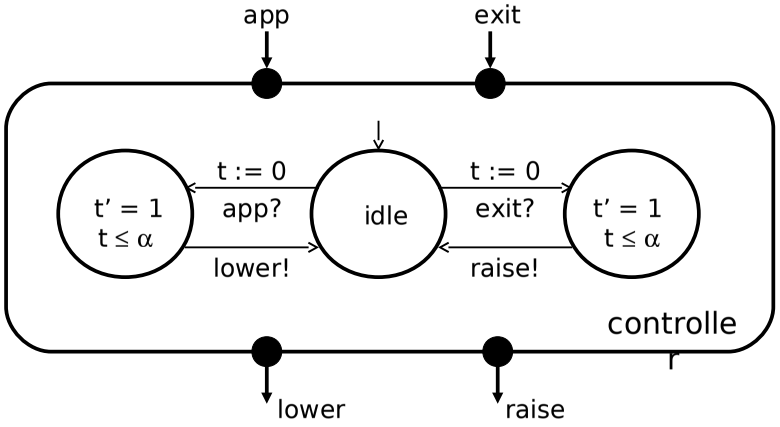
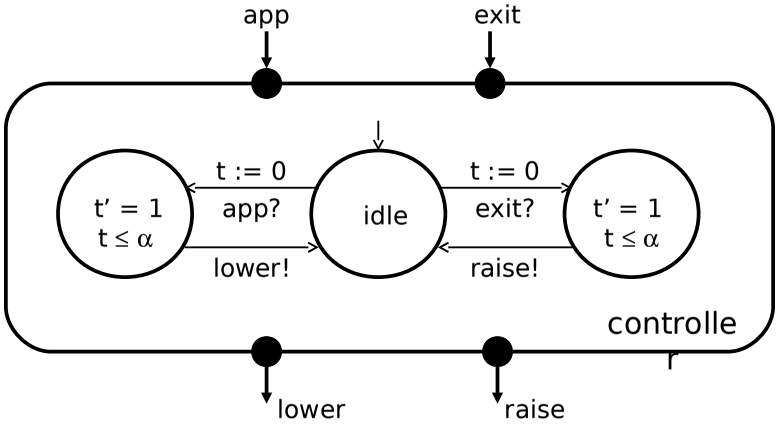


y: uninitialized singular variable





t: clock
α: parameter

Properties

Safety: $\forall\Box\ (\ x \le 10\ \Rightarrow\ loc[gate] = closed\ )$

"on all trajectories, always"

For which values of α is this true?

Properties

Safety: $\forall\Box\ (\ x \le 10\ \Rightarrow \mathrm{loc[gate]} = \mathrm{closed}\ )$

Liveness: $\forall\Box\ \forall\Diamond(\ \mathrm{loc[gate]} = \mathrm{open}\ )$

"on all trajectories, eventually"

Properties

Safety: $\forall\Box\ (\ x \le 10\ \Rightarrow \mathrm{loc[gate]} = \mathrm{closed}\ )$

Liveness: $\forall\Box\ \forall\Diamond\ (\ \mathrm{loc[gate]} = \mathrm{open}\ )$

Real time: $\forall\Box\ z := 0.\ (\ z' = 1\ \Rightarrow \forall\Diamond\ (\ \mathrm{loc[gate]} = \mathrm{open}\ \wedge\ z \le 60\ ))$

clock variable

Properties

Safety: $\forall\Box\ (\ x \le 10\ \Rightarrow \mathrm{loc[gate]} = \mathrm{closed}\ )$

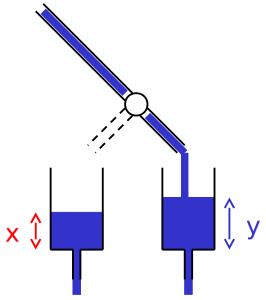
Liveness: $\forall\Box\ \forall\Diamond\ (\ \mathrm{loc[gate]} = \mathrm{open}\ )$

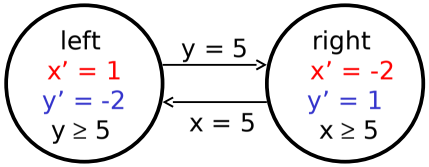
Real time: $\forall\Box\ z := 0.\ (\ z' = 1\ \Rightarrow \forall\Diamond\ (\ \mathrm{loc[gate]} = \mathrm{open}\ \wedge\ z \le 60\ ))$

Nonzeno: $\forall\Box\ z := 0.\ (\ z' = 1\ \Rightarrow\ \exists\Diamond\ (\ z = 1\ ))$

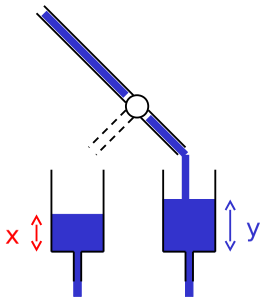
"on some trajectory, eventually"

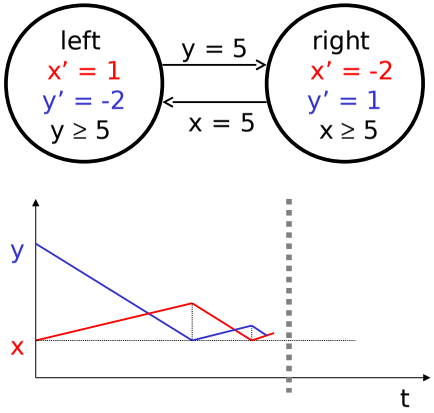
A Zeno System

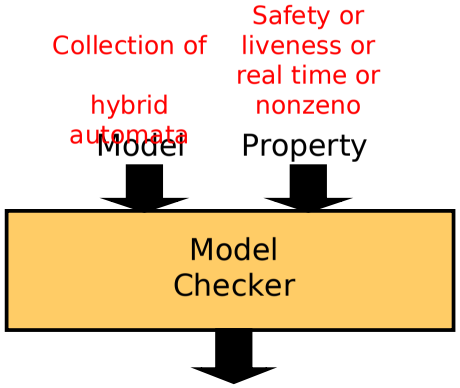


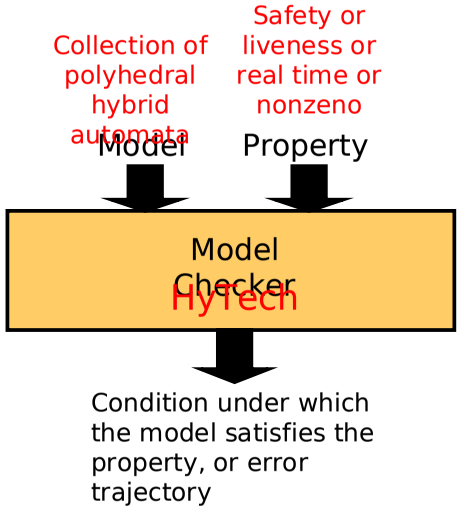


A Zeno System

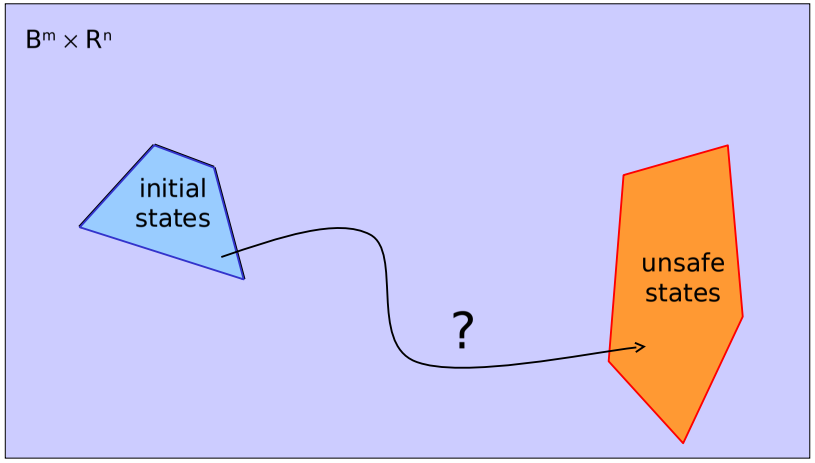








Model Checking for Safety



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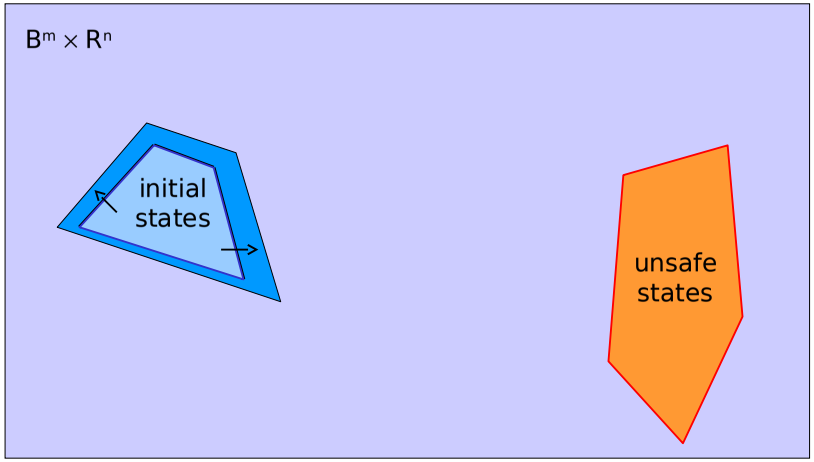

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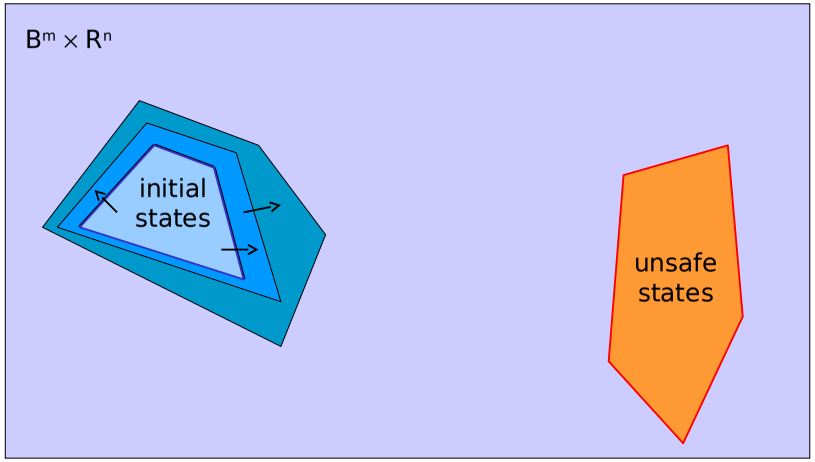

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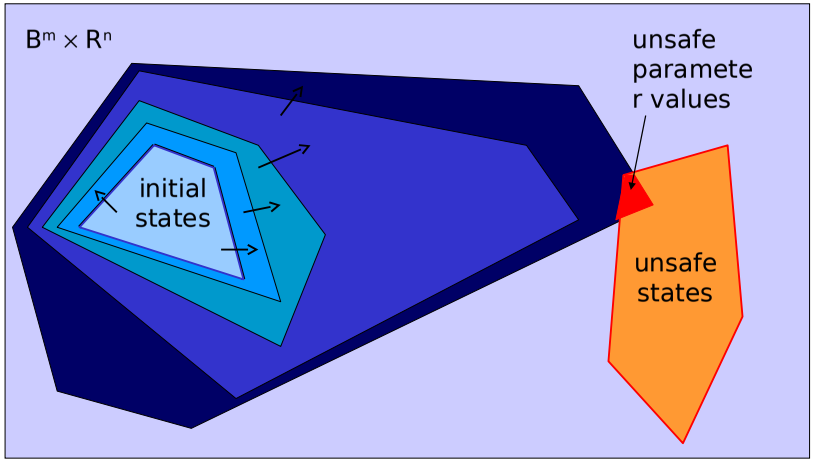

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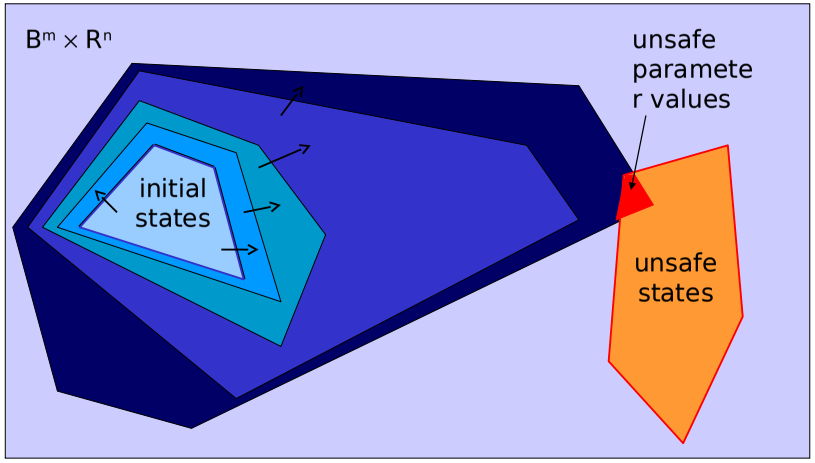

This is guaranteed to terminate if all variables are initialized (e.g. clocks).

Applications of HyTech and Derivations:
polyhedral
overapproximation of dynamics

-automotive engine control [Wong-Toi et al.]
-chemical plant control [Preussig et al.]
-flight control [Honeywell; Rockwell-Collins]
-air traffic control [Tomlin et al.]
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Successor Tools:

1. More expressive region algebras, e.g. FO(R,≤, +,·) still permits quantifier elimination [Pappas et al.]

2. Different approximations, e.g. ellipsoid regions instead of polyhedral regions [Varaiya et al.]

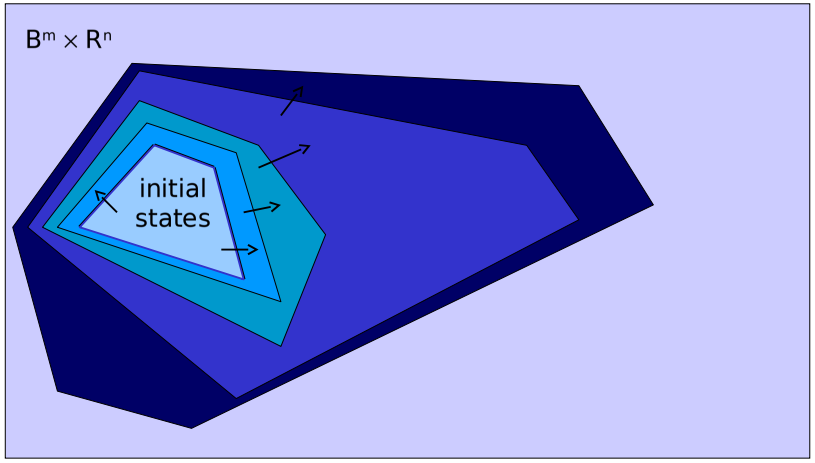
Symbolic Transition System

Q
Σ
pre, post
A
$\Re$

Region algebra:

1. A $\subseteq \Re$
2. pre, post: $\Re \times \Sigma \to \Re$ computable
3. Å : $\Re^2 \to \Re$
\ : $\Re^2 \to \Re$
computable
$\subseteq$: $\Re^2 \to \{ t, f \}$

Symbolic Reachability Analysis



Symbolic Semi-Algorithms

Starting from the observations in A, compute new regions in $\Re$ by applying the operations pre, post, Å , \ , and $\subseteq$.

Termination?

Five Verification Questions

V1: Reachability $\exists\Diamond$ b
Is an invariant always true? $\exists\Diamond$ unsafe

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Conjunction-free μ-calculus
Is an invariant true every other step?

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Is an invariant true every other step?

V3: Repeated Reachability $\exists\Box\Diamond$ b
Linear temporal logic (LTL)
Liveness $\exists(\Box\Diamond$ fair $\wedge \neg\Diamond$ goal)

Five Verification Questions

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Is an invariant always true? $\exists\Diamond$ unsafe

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V4: Nested Reachability $\exists\Diamond\ (b \wedge \exists\Diamond\ b_1 \wedge \exists\Diamond\ b_2)$
Half branching temporal logic ($\exists$CTL, $\forall$CTL)

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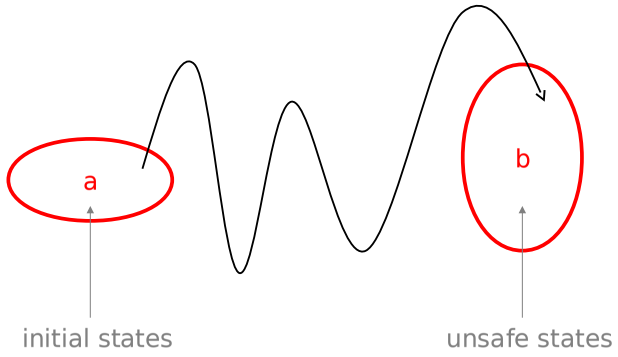
Half branching temporal logic ($\exists$CTL, $\forall$CTL)

V5: Negated Reachability $\forall\Box\ (b \rightarrow \exists\Diamond\ c)$ Full branching temporal logic (CTL)
Nonzenoness $\forall\Box\ (tick \rightarrow pre(\exists\Diamond\ tick))$

V1: Symbolic Reachability

$a \wedge \exists\Diamond\ b$

Given a,b$\in$A, is there a trajectory from a to b?



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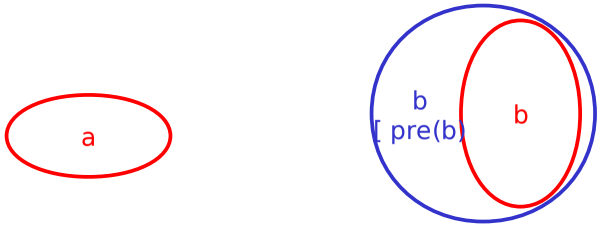

$pre(b) = \cup_{\sigma\in\Sigma} pre(b,\sigma)$

V1: Symbolic Reachability

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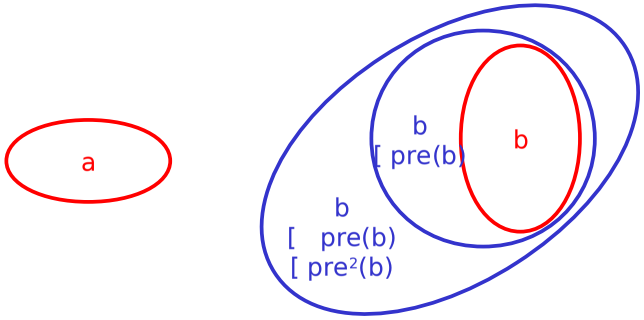

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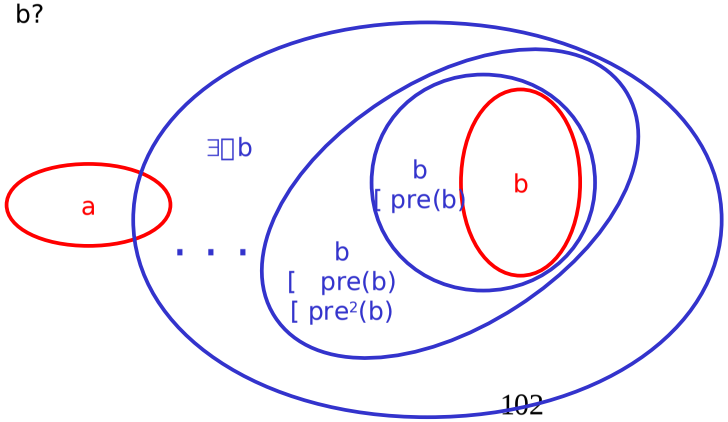

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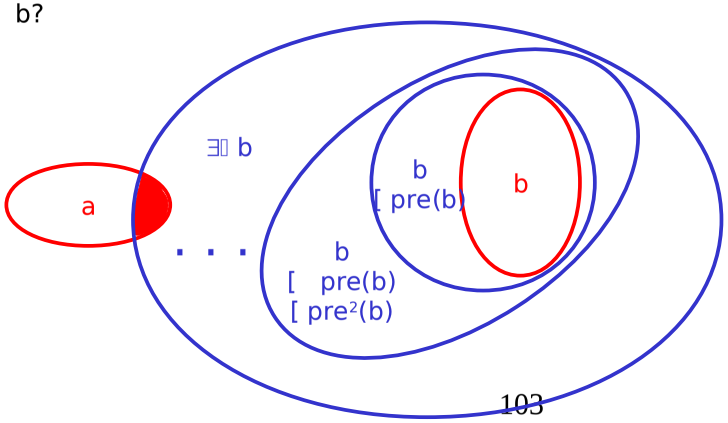

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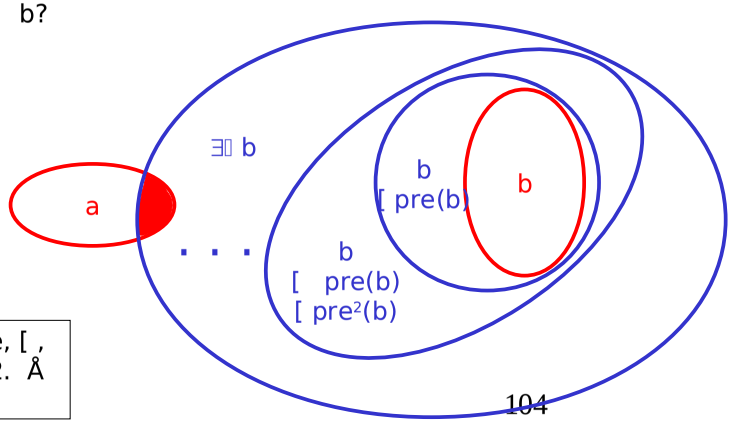

1. pre, [,
⊆ 2. Å
a

V2: Symbolic Counting Reachability

$a \wedge \mu X.\ (b \vee pre^2(X))$

Given a,b∈A, is there a trajectory of even length from a to b?

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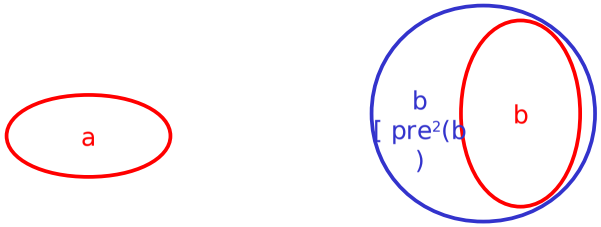
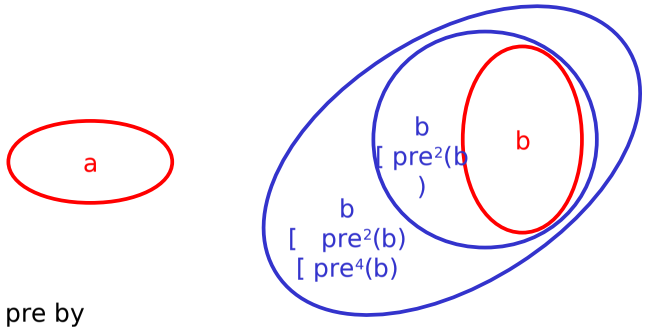


Replace pre by pre^2 in reachability.

V2: Symbolic Counting Reachability

$a \wedge \mu X.\ (b \vee pre^2(X))$

Given a,b∈A, is there a trajectory of even length from a to b?



Replace pre by pre^2 in reachability.

V2: Symbolic Counting Reachability

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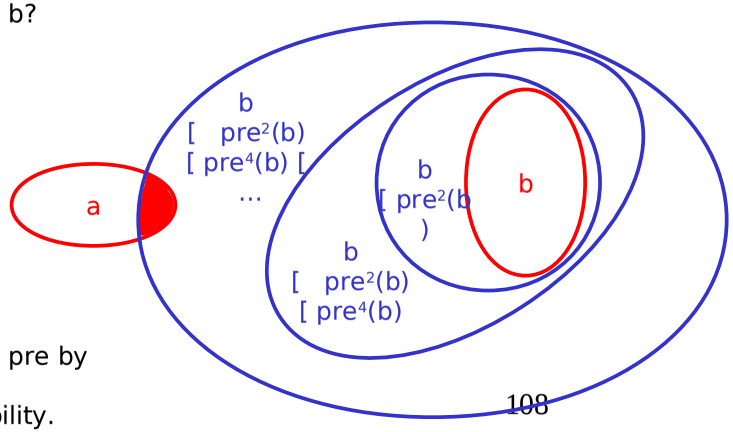

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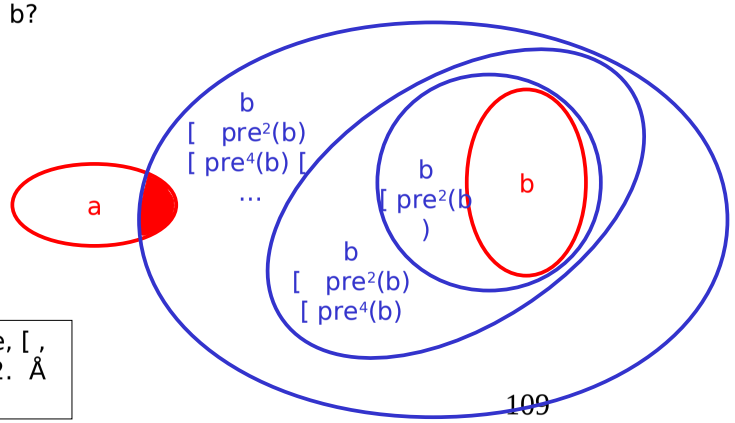

1. pre, [, ⊆ 2. Å
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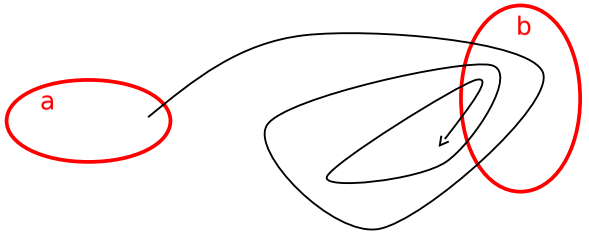
V3: Symbolic Repeated Reachability

$a \wedge \exists\Box\, b$

Given a,b∈A, is there an infinite trajectory from a that visits b infinitely often?



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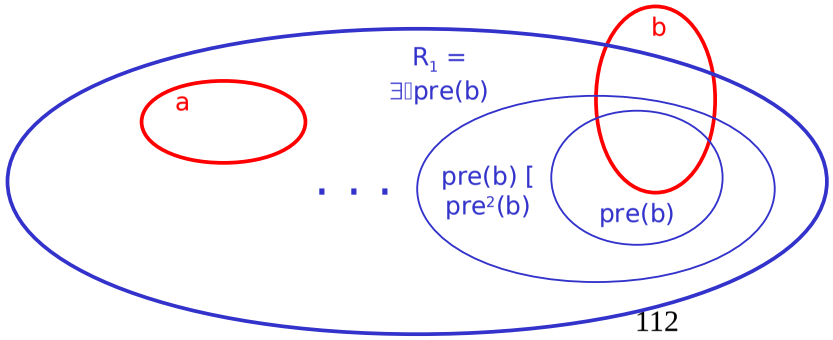

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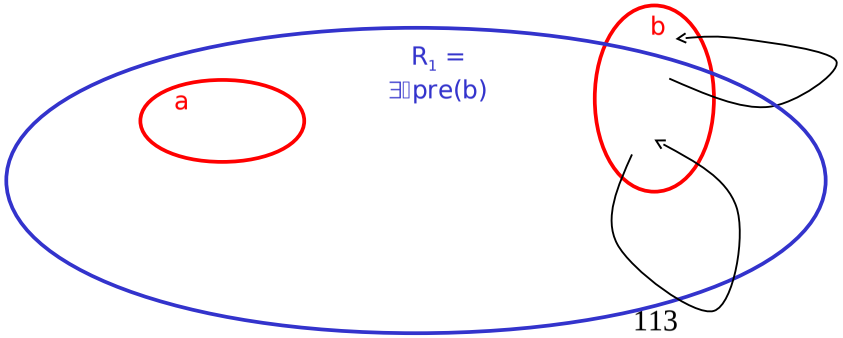

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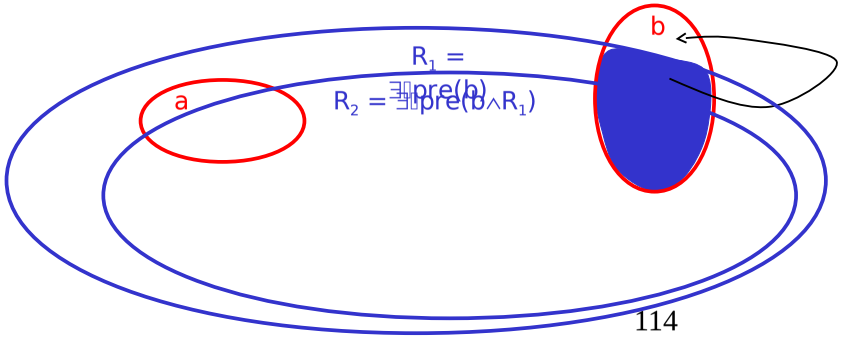

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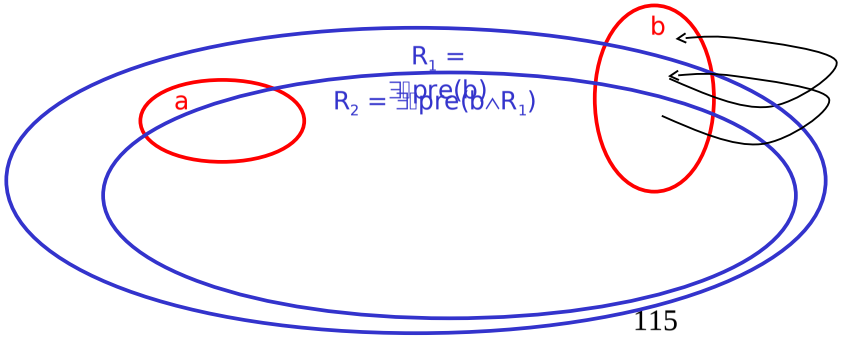

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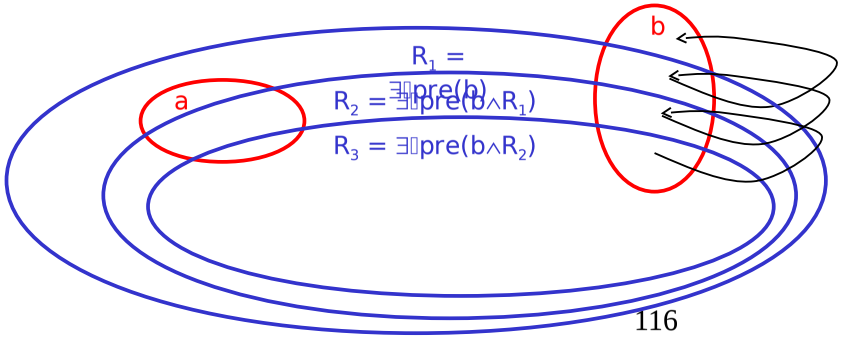

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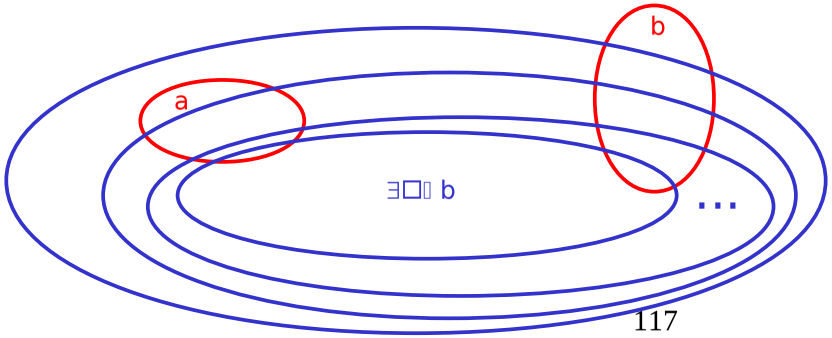

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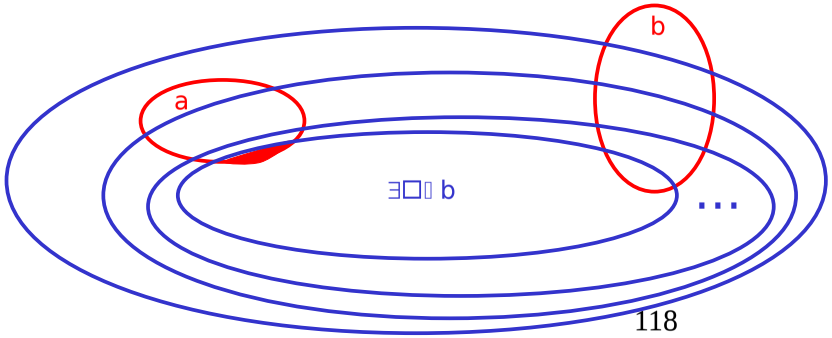

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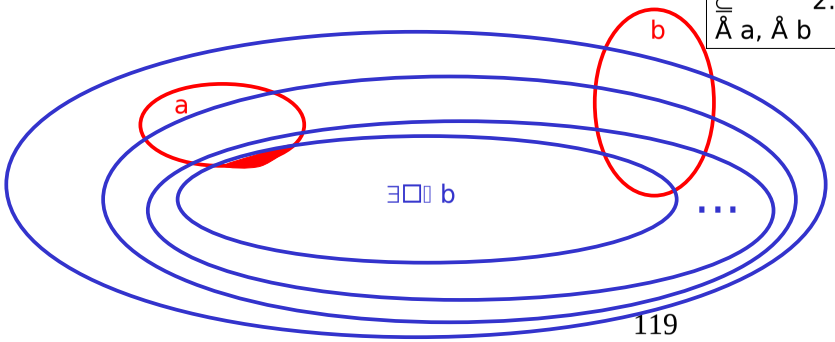
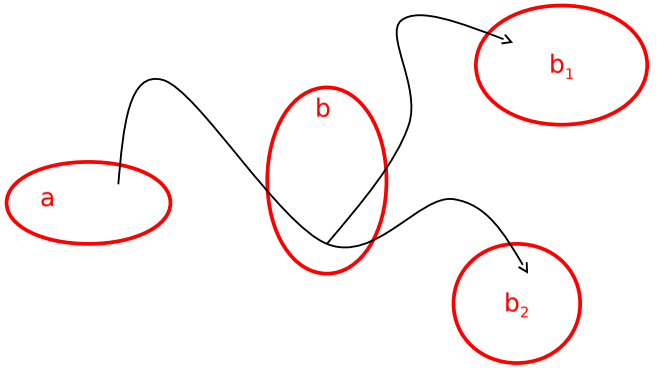


V4: Symbolic Nested Reachability

$a \wedge \exists\Diamond\, (b \wedge \exists\Diamond\, b_1 \wedge \exists\Diamond\, b_2)$



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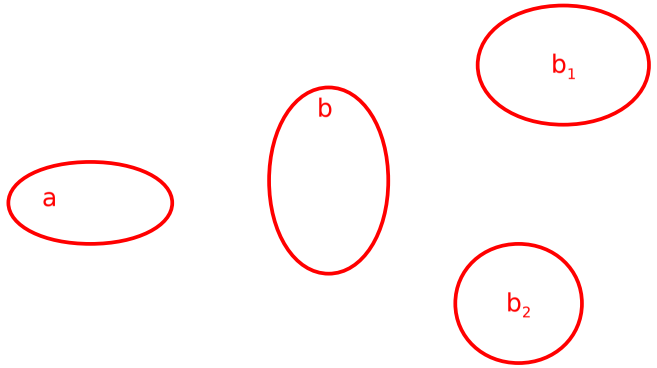

121

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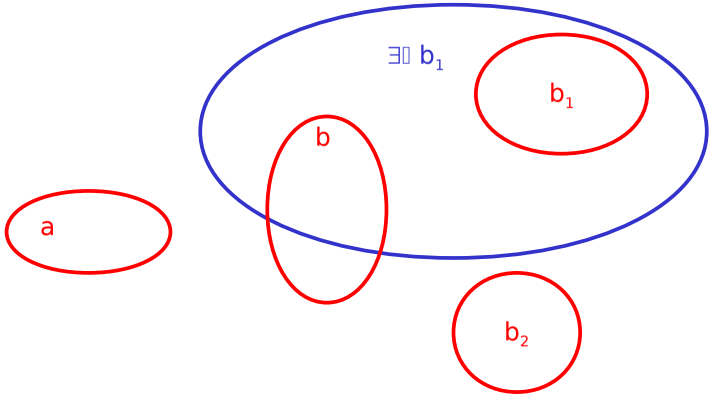
$a \wedge \exists\Box\, (b \wedge \exists\Box\, b_1 \wedge \exists\Box\, b_2)$



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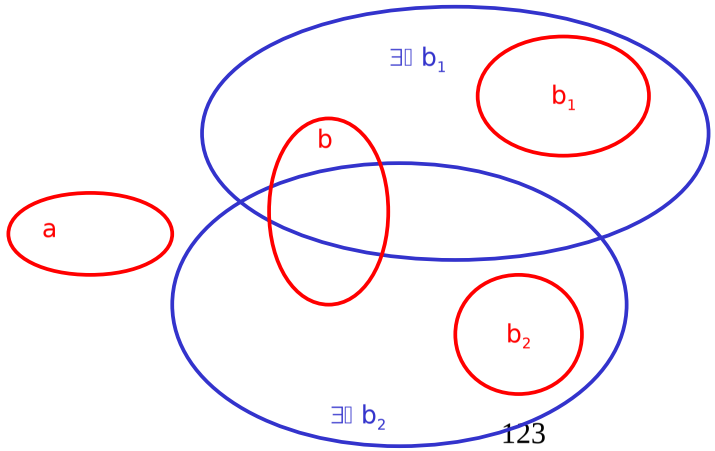

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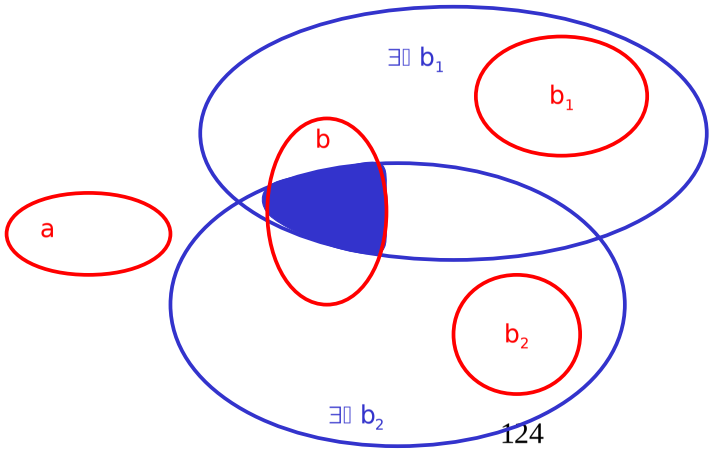
$a \wedge \exists\Box\, (b \wedge \exists\Box\, b_1 \wedge \exists\Box\, b_2)$



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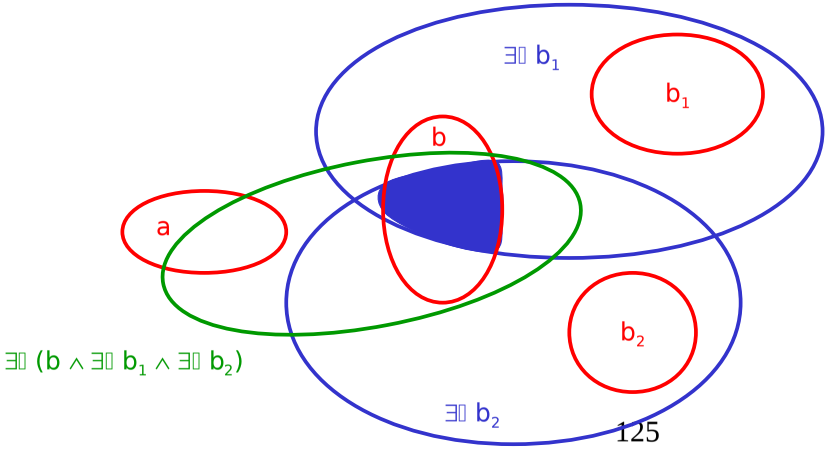

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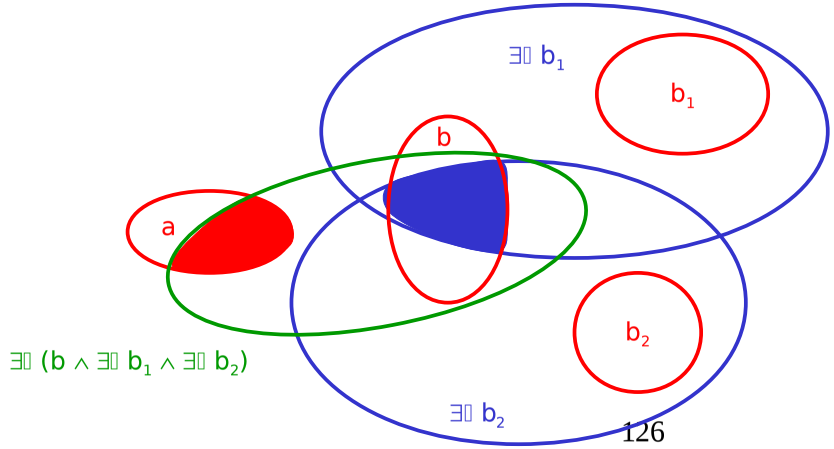

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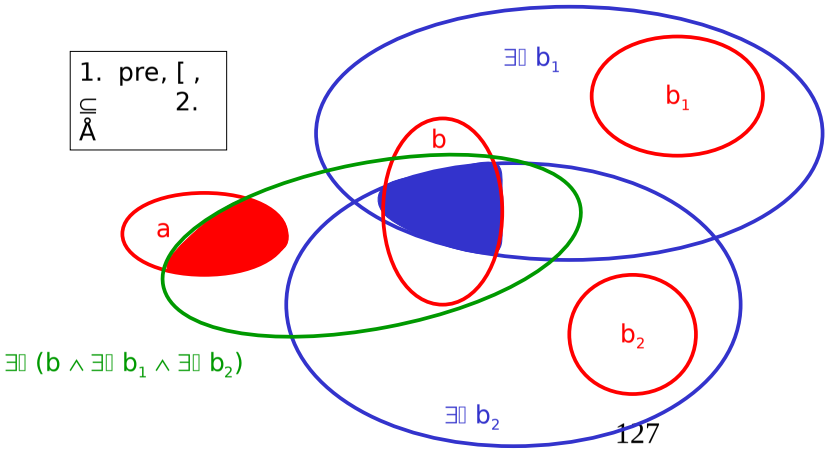
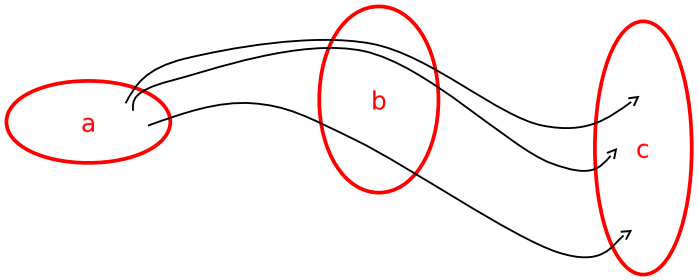


V5: Symbolic Negated Reachability

$a \wedge \forall\Box\, (b \rightarrow \exists\Box\, c)$

Given a,b,c∈A, can every trajectory from a to b be extended to c?



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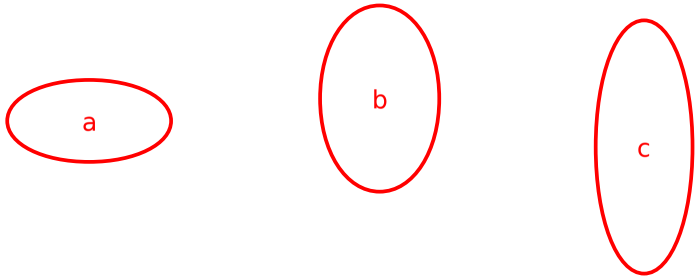

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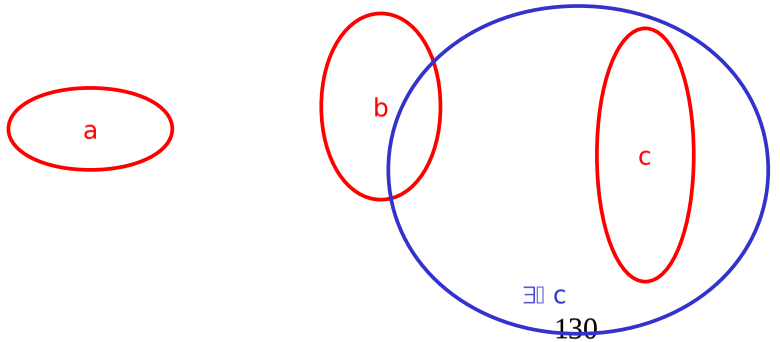

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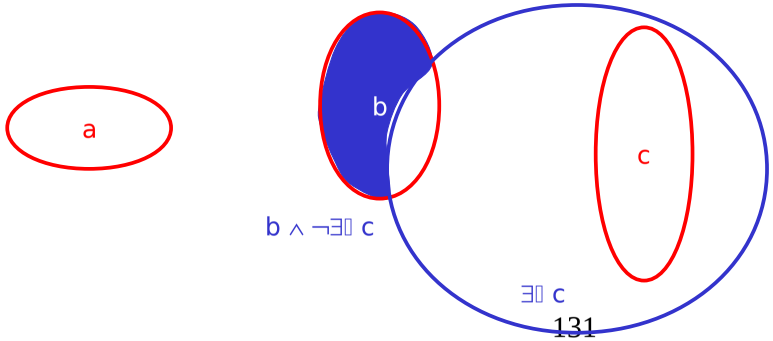

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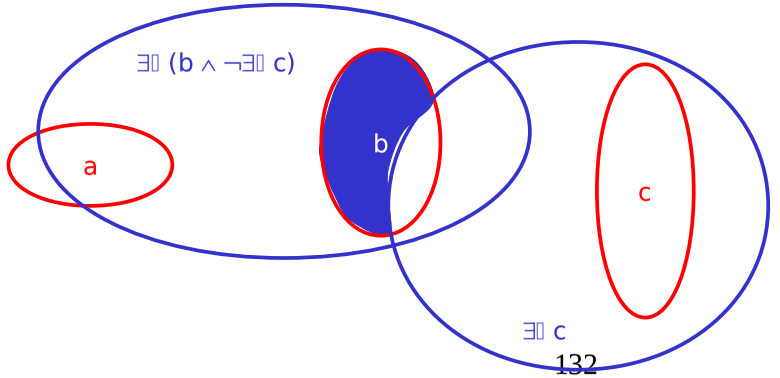

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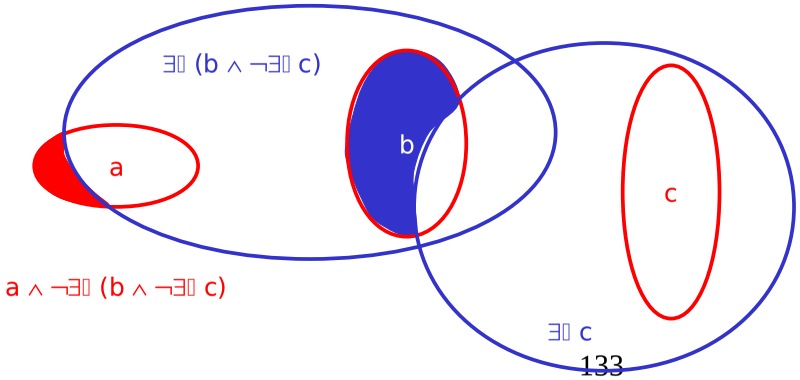

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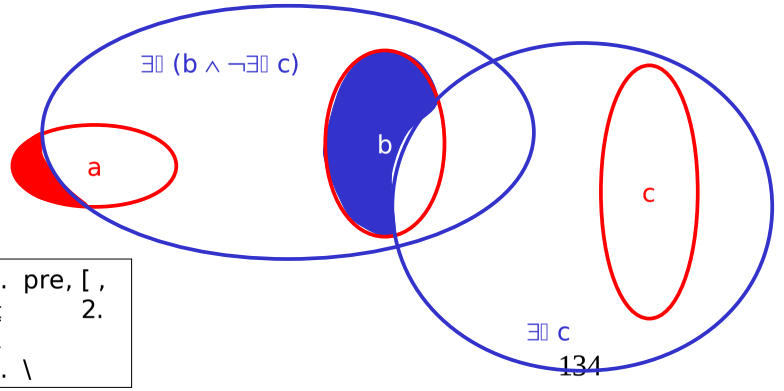

1. pre, [, ⊆ 2. Å
3. \

Five Specification Logics

L1: Reachability Logic

φ := a | φ∨φ | ∃φ

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φ := a | φ∨φ | ∃φ

L2: Conjunction-free μ-Calculus

φ := a | X | φ∨φ | pre(φ) | μX.φ
Symbolic model checking: pre, [, ⊆

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L3: Guarded μ-Calculus (subsumes LTL, omega automata)

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L4: Existential μ-Calculus (subsumes ∃CTL)

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Five Specification Logics

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Symbolic model checking: pre, [, ⊆ , Å a

L4: Existential μ-Calculus (subsumes ∃CTL)

φ := a | X | φ∨φ | φ∧φ | pre(φ) | μX.φ | νX.φ
Symbolic model checking: pre, [, ⊆ , Å

L5: μ-Calculus (subsumes CTL)

φ := a | X | φ∨φ | φ∧φ | pre(φ) | pre(φ) | μX.φ | νX.φ
Symbolic model checking: pre, [, ⊆ , Å , \

pre(φ)=: pre:(φ)

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

for each a∈A do
R_0 := a
for i=1,2,3,...
do
$R_i := R_{i-1}$ [pre(R_{i-1}) until $R_i = R_{i-1}$

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

A2: Close A under pre

$\Im_0 := A$
for i=1,2,3,... do
$\Im_i := \Im_{i-1} [\{ pre(R) \mid R \in \Im_i \}$

until $\Im_i = \Im_{i-1}$

$A = \{a_1, a_2\}$

A1 computes: $a_1[pre(a_1)$,
$a_1[pre(a_1) [pre^2(a_1)$,
$a_1[pre(a_1) [pre^2(a_1) [pre^3(a_1)$, ...

$a_2[pre(a_2)$,
$a_2[pre(a_2) [pre^2(a_2)$, ...

A2 computes: $pre(a_1)$, $pre^2(a_1)$, $pre^3(a_1)$, ...
$pre(a_2)$, $pre^2(a_2)$, $pre^3(a_2)$, ...

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

A2: Close A under pre

A3: Close A under pre, Å a

$\Im_0 := A$
for i=1,2,3,... do
$\Im_i := \Im_{i-1} [\{ pre(R) \mid R \in \Im_i \} [\{ R Å a \mid R \in \Im_i, a \in A \}$

until $\Im_i = \Im_{i-1}$

$A = \{a_1, a_2\}$

A1 computes: $a_1[pre(a_1)$,
$a_1[pre(a_1) [pre^2(a_1)$,
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A2 computes: $pre(a_1)$, $pre^2(a_1)$, $pre^3(a_1)$, ...
$pre(a_2)$, $pre^2(a_2)$, $pre^3(a_2)$, ...

A3 computes: also $pre(a_1) Å a_2$ etc.

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

A2: Close A under pre

A3: Close A under pre, Å a

A4: Close A under pre, Å

$\Im_0 := A$
  for i=1,2,3,... do
    $\Im_i := \Im_{i-1}$
[{ pre(R) | R∈$\Im_i$ } ∪
  [{ R_1 Å R_2 | R_1,R_2∈$\Im_i$ } ∪

until $\Im_i = \Im_{i-1}$

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

A2: Close A under pre

A3: Close A under pre, Å a

A4: Close A under pre, Å

A5: Close A under pre, Å , \

$\Im_0 := A$
  for i=1,2,3,... do
    $\Im_i := \Im_{i-1}$
[{ pre(R) | R∈$\Im_i$ } ∪
  [{ R_1 Å R_2 | R_1,R_2∈$\Im_i$ } ∪
    [{ R_1\ R_2 | R_1,R_2∈$\Im_i$ }
until $\Im_i = \Im_{i-1}$

Five Symbolic Semi-Algorithms

A1: Symbolic backward reachability

A2: Close A under pre

A3: Close A under pre, Å a

A4: Close A under pre, Å

A5: Close A under pre, Å , \

A_k terminates ($1 \le k \le 5$) ⇒
symbolic model checking of L_k terminates.

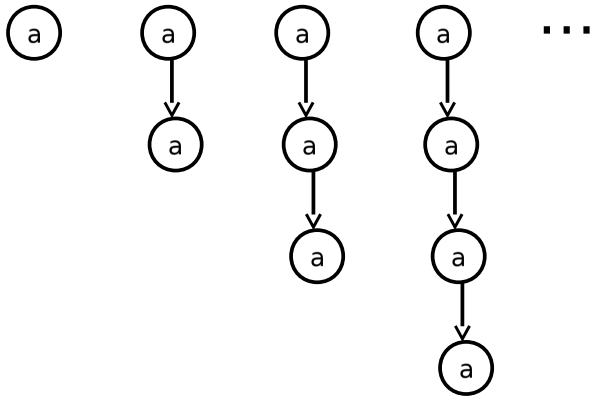
Five State Equivalences

E1: Bounded-Reach Equivalence

$q_1 \cong_1 q_2$ iff if a∈A can be reached from q_1 in d steps, then a can be reached from q_2 in at most d steps, and vice versa.

E2: Distance Equivalence

$q_1 \cong_2 q_2$ iff if a∈A can be reached from q_1 in d steps, then a can be reached from q_2 in d steps, and vice versa.



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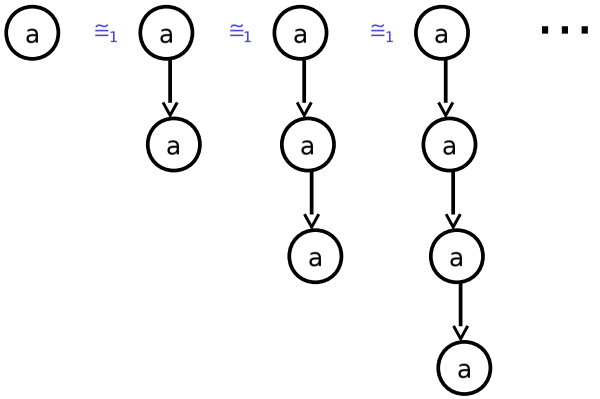

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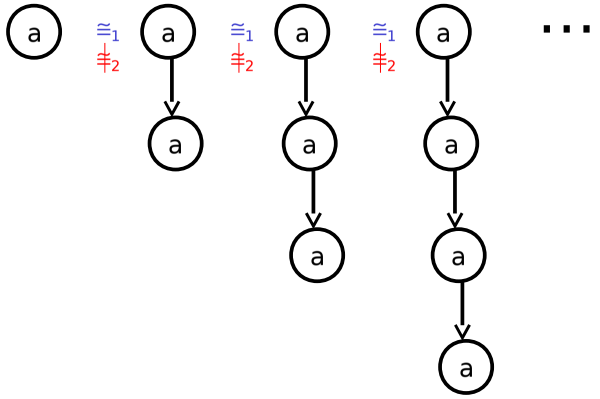

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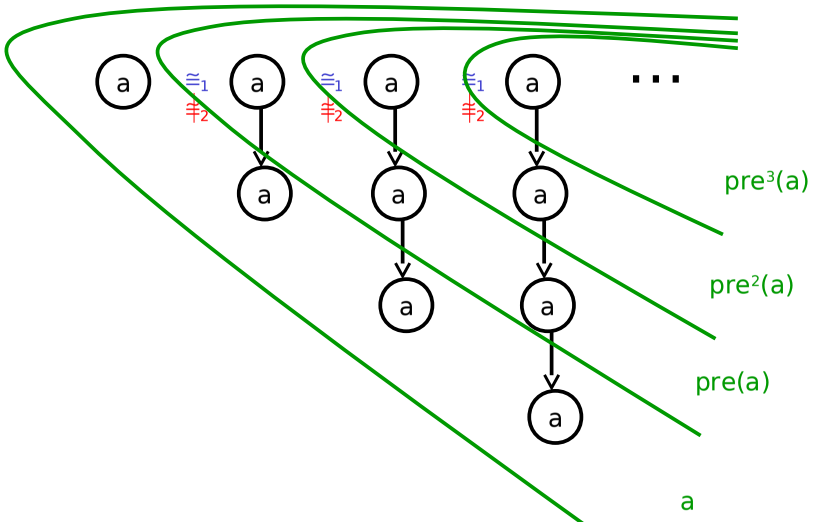

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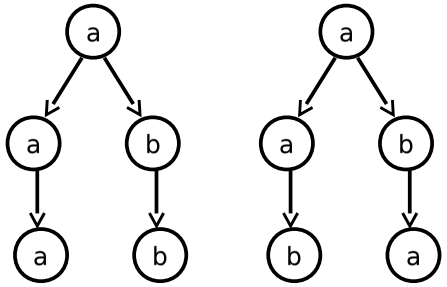
Five State Equivalences

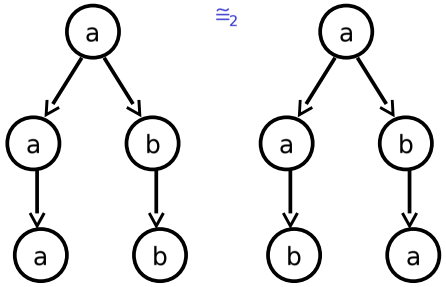
E1: Bounded-Reach Equivalence

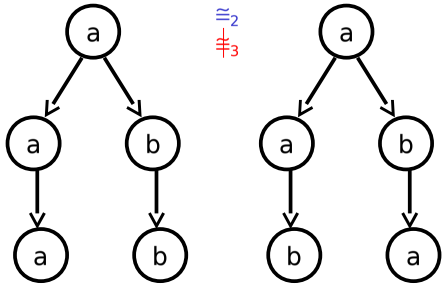
E2: Distance Equivalence

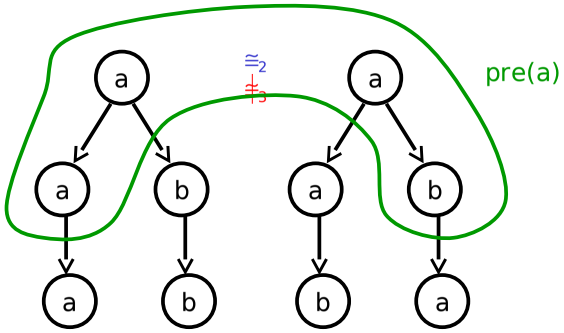
E3: Trace Equivalence

$q_1 \cong_3 q_2$ iff if every finite trace from q_1 is a finite trace from q_2, and vice versa.









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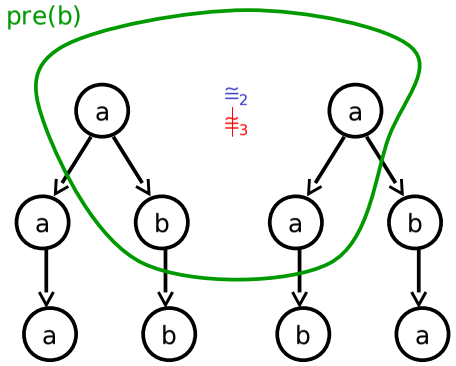

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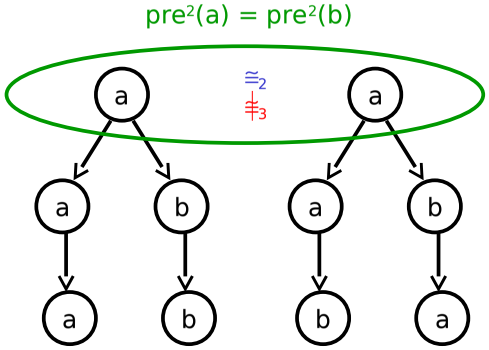

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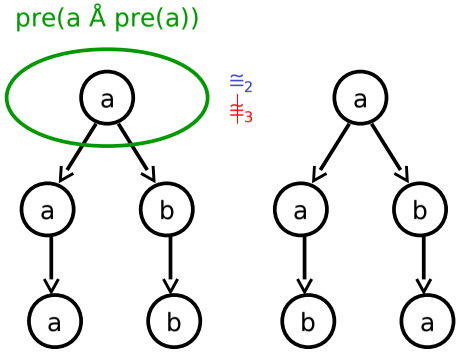


160

Five State Equivalences

E1: Bounded-Reach Equivalence

E2: Distance Equivalence

E3: Trace Equivalence

E4: Similarity (mutual simulation)

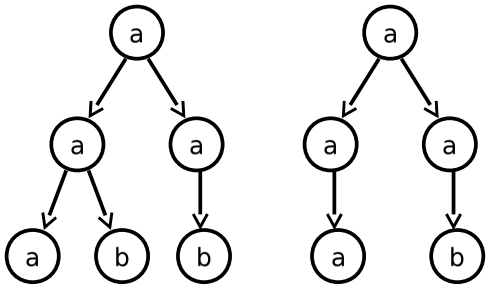
$q_1 \cong_4 q_2$ iff if q_1 simulates q_2, and vice versa.

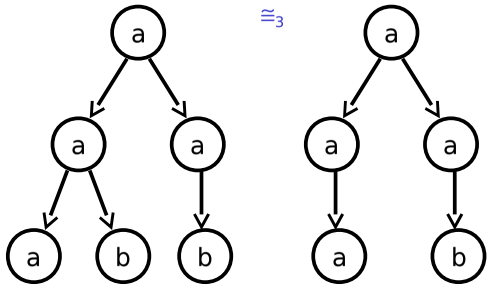
q_1 is simulated by q_2

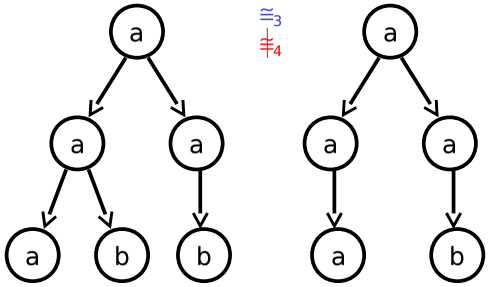
iff

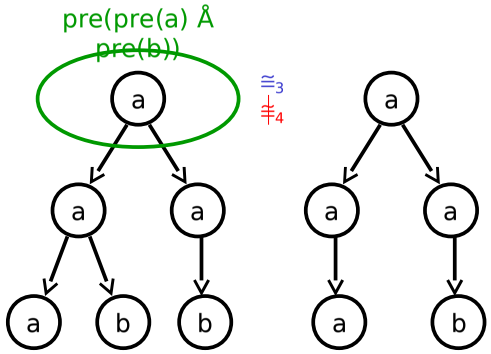
there is a simulation relation S such that

1. $S(q_1,q_2)$
2. if S(p,q) then
   a. (8 a2A) (p2 a iff q2 a)
   b. (8 p') (if p2 pre(p') then
      (9 q') (q2 pre(q') Æ S(p',q')))









Five State Equivalences

E1: Bounded-Reach Equivalence

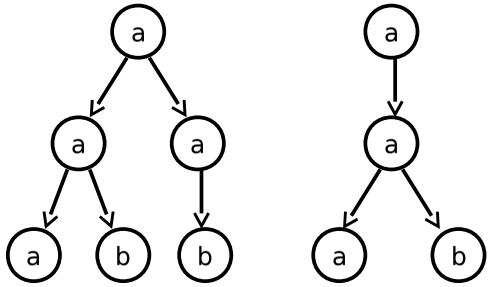
E2: Distance Equivalence

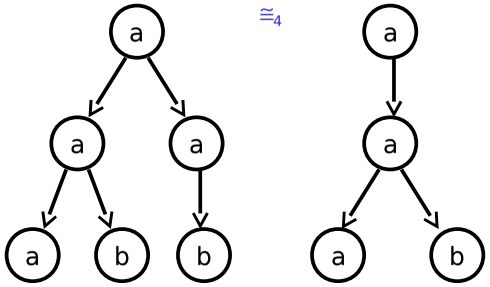
E3: Trace Equivalence

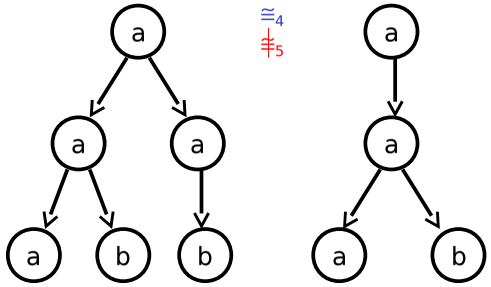
E4: Similarity (mutual simulation)

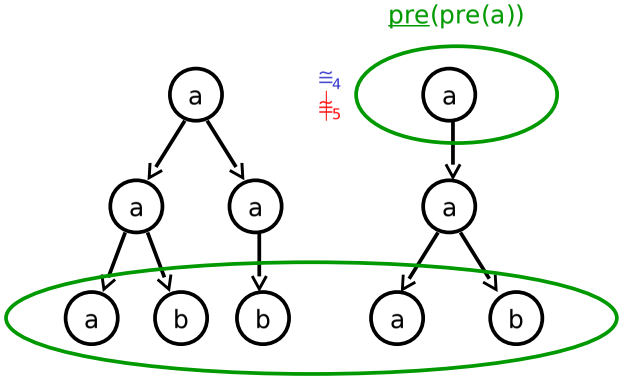
E5: Bisimilarity

$q_1 \cong_5 q_2$ iff if q_1 simulates q_2 via a symmetric simulation relation (this is called a bisimulation relation).









Specification Logics:

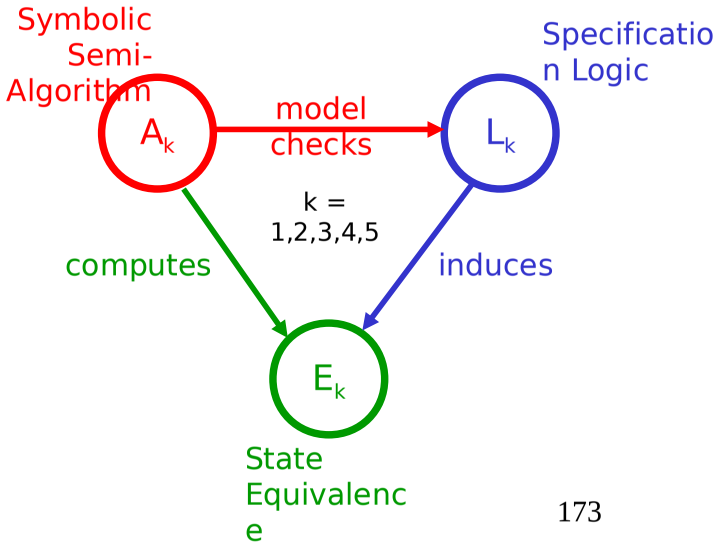
L1	Reachability	
L2	Conjunction-free μ-calculus	
L3	Guarded μ-calculus / LTL / omega automata	
L4	Existential μ-calculus / ∃CTL	
L5	μ-Calculus / CTL	

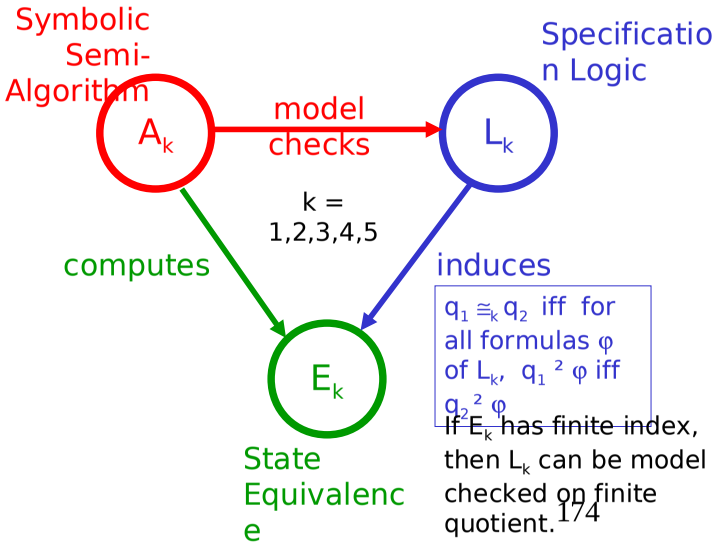
State Equivalences:

E1	Bounded-reach equivalence
E2	Distance equivalence
E3	Trace equivalence
E4	Similarity
E5	Bisimilarity

Symbolic Semi-Algorithms:

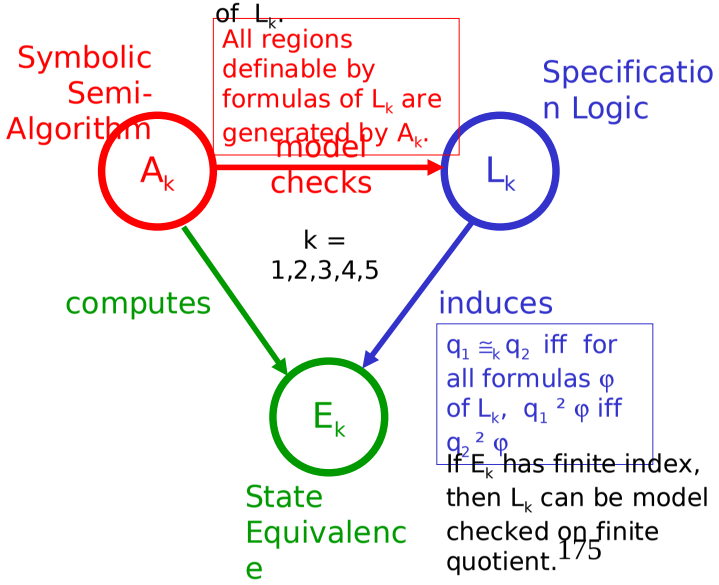
A1	Backwards pre iteration
A2	Closure under pre
A3	Closure under pre, Å a
A4	Closure under pre, Å
A5	Closure under pre, Å , \ ("partition refinement")





If E_k has finite index, then L_k can be model checked on finite quotient.

A_k terminates iff symbolic model checking terminates for all formulas of L_k.



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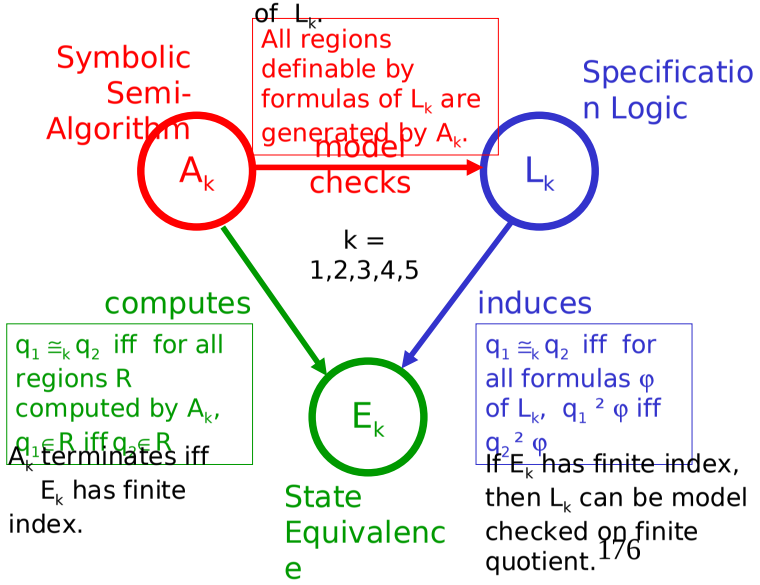

A_k terminates iff E_k has finite index.

If E_k has finite index, then L_k can be model checked on finite quotient.

Five Classes of Symbolic Transition Systems

STS1: pre^* terminates ⇔ Finite bounded-reach equiv ⇒ ∃□ decidable

STS2: pre closure terminates ⇔ Finite distance equiv ⇒ conjunction-free μ-calculus decidable

STS3: (pre, Å a) closure terminates ⇔ Finite trace equiv ⇒ guarded μ-calculus (LTL, omega automata) decidable

STS4: (pre, Å) closure terminates ⇔ Finite similarity ⇒ existential μ-calculus (∃CTL, ∀CTL) decidable

STS5: (pre, Å , \ ) closure terminates ⇔ Finite bisimilarity ⇒ μ-calculus (CTL) decidable

Five Classes of Symbolic Transition Systems

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STS2: pre closure terminates ⇔ Finite distance equiv ⇒ conjunction-free μ-calculus decidable

STS3: (pre, Å a) closure terminates ⇔ Finite trace equiv ⇒ guarded μ-calculus (LTL, omega automata) decidable Initialized rectangular hybrid automata

STS4: (pre, Å) closure terminates ⇔ Finite similarity ⇒ existential μ-calculus (∃CTL, ∀CTL) decidable
2D initialized rectangular hybrid automata

STS5: (pre, Å , \ ) closure terminates ⇔ Finite bisimilarity ⇒ μ-calculus (CTL) decidable
Initialized singular hybrid automata

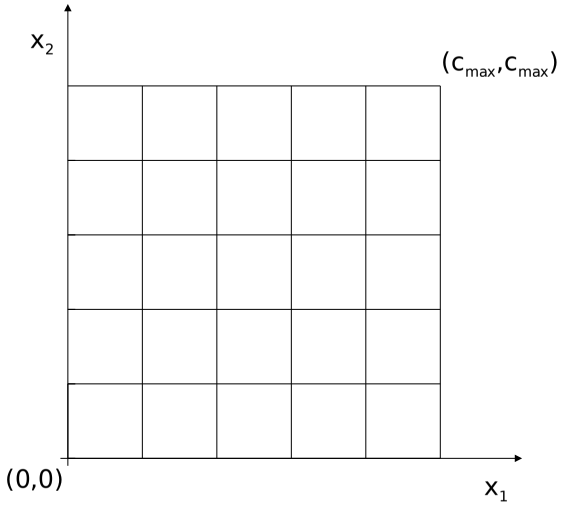
Example: Singular Hybrid Automata

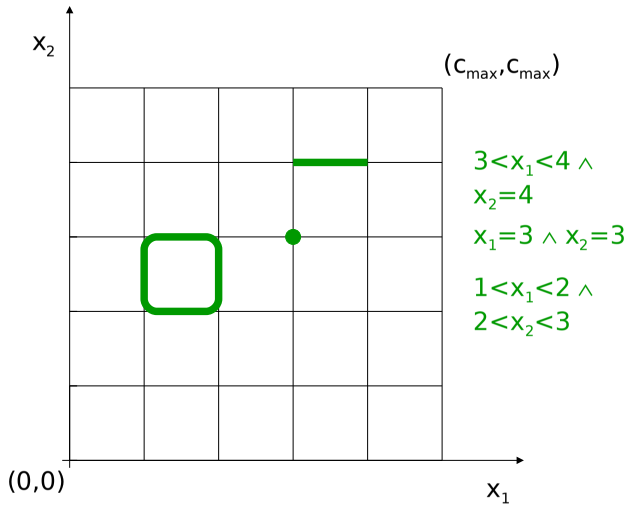
$Q = B^m \times R^n$

Invariants and guards:
integral bounds, e.g. $x_1 < 7 \wedge 1 \leq x_2 \leq 2$

Flows: constant slopes, e.g. $x'_1 = 1;\ x'_2 = 2$ Jumps: integral assignments, e.g. $x_1 := 0;\ x_2 := 5$

$A = \{ x_i = c,\ c < x_i < c+1 \mid 1 \leq i \leq n,\ c \in N,\ c < c_{max} \}$





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Initialized: assignment when slope changes.

Special Case: Timed Automata

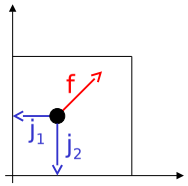
$Q = B^m \times R^n$

Invariants and guards:
integral bounds, e.g. $x_1 < 7 \wedge 1 \leq x_2 \leq 2$

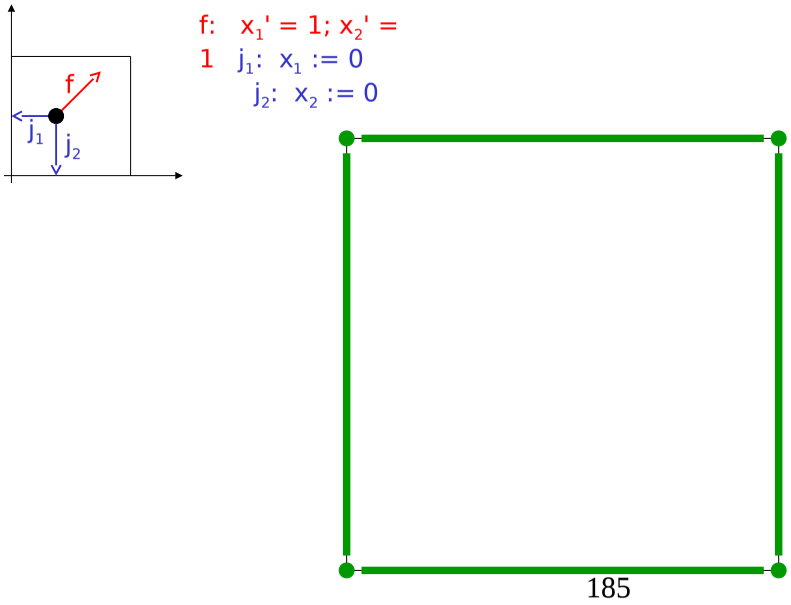
Flows: clocks, e.g. $x'_1 = 1;\ x'_2 = 1$
Jumps: integral assignments, e.g. $x_1 := 0;\ x_2 := 5$

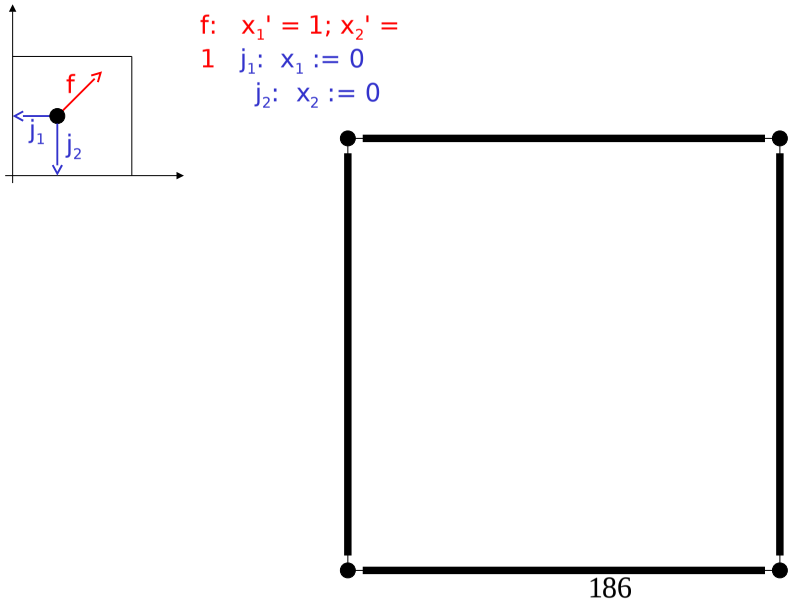
$A = \{ x_i = c, c < x_i < c+1 \mid 1 \leq i \leq n, c \in N, c < c_{max} \}$

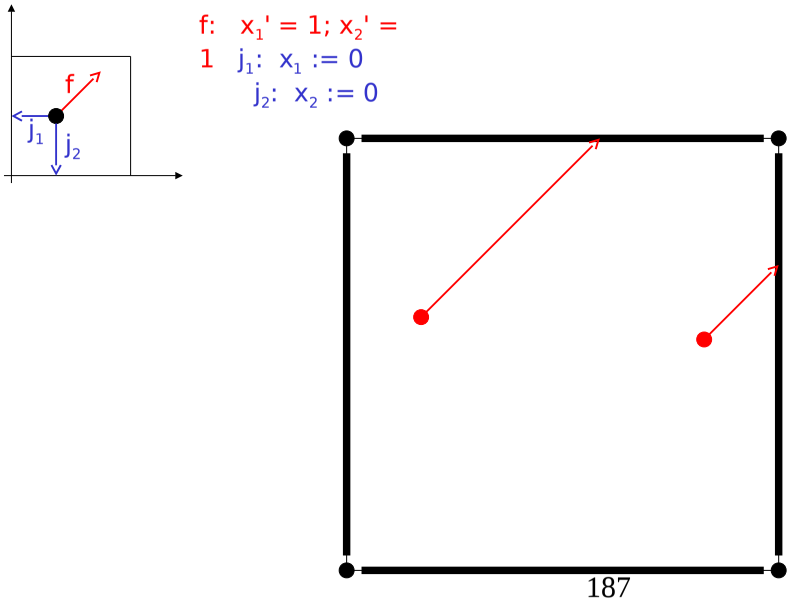
Always initialized.

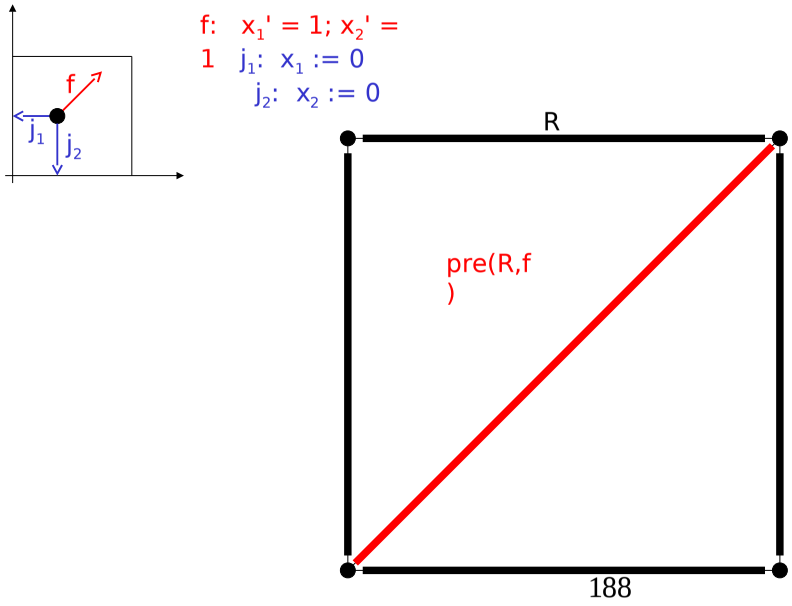


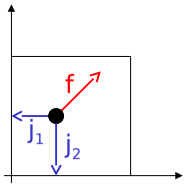
f: $x_1' = 1; x_2' = 1$
j_1: $x_1 := 0$
j_2: $x_2 := 0$


f: x1' = 1; x2' =
1 j1: x1 := 0
j2: x2 := 0
f
j1
j2
185

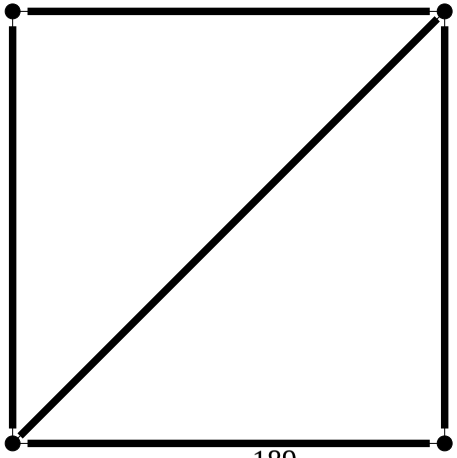

f: x1' = 1; x2' =
1 j1: x1 := 0
j2: x2 := 0
f
j1
j2
186

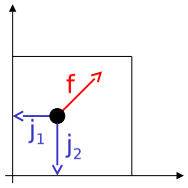

f: x1' = 1; x2' =
1 j1: x1 := 0
j2: x2 := 0
f
j1
j2
187


f: x1' = 1; x2' =
1 j1: x1 := 0
j2: x2 := 0
f
j1
j2
R
pre(R,f
)
188



f: $x_1' = 1; x_2' = 1$
j_1: $x_1 := 0$
j_2: $x_2 := 0$





f: $x_1' = 1; x_2' = 1$
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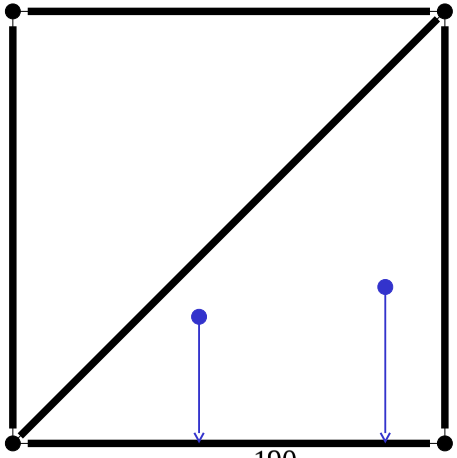

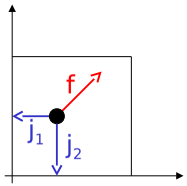

f: $x_1' = 1; x_2' = 1$
j_1: $x_1 := 0$
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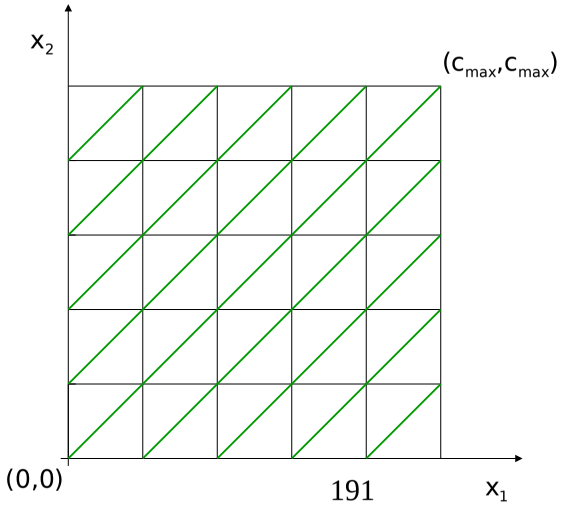


Finite bisimulation.

Timed Automata

Clock regions:

boolean combinations of

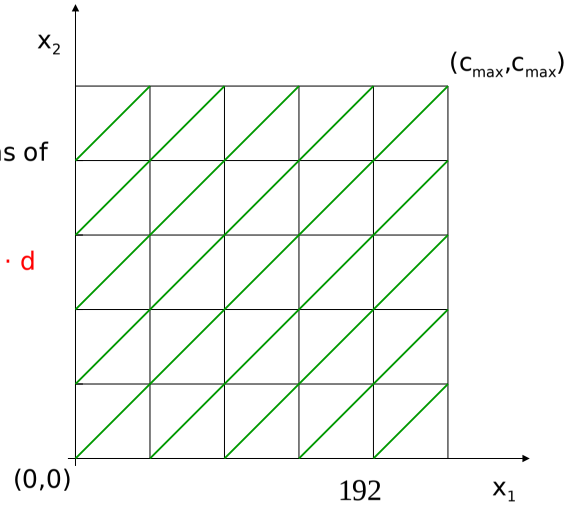
$x_1 \cdot c$

$x_1 - x_2 \cdot d$

(clock difference formulas)

Finite bisimulation.

[Alur, Dill]



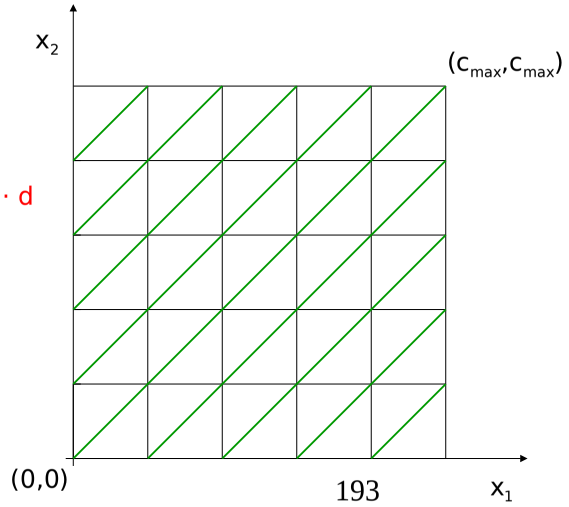
Timed Automata

Corollary:
guards and invariants

$x_1 - x_2 \cdot d$

are permissible.

Finite bisimulation.

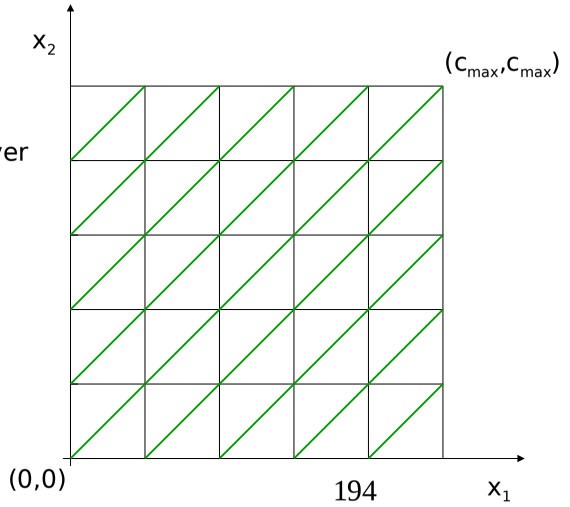


Timed Automata

Corollaries:

CTL model checking over timed automata is decidable.

Finite bisimulation.



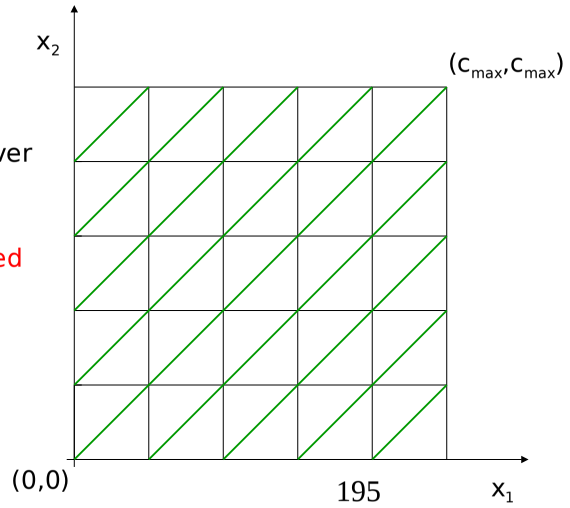
Timed Automata

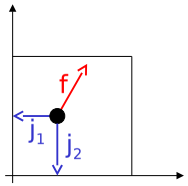
Corollaries:

CTL model checking over timed automata is decidable.

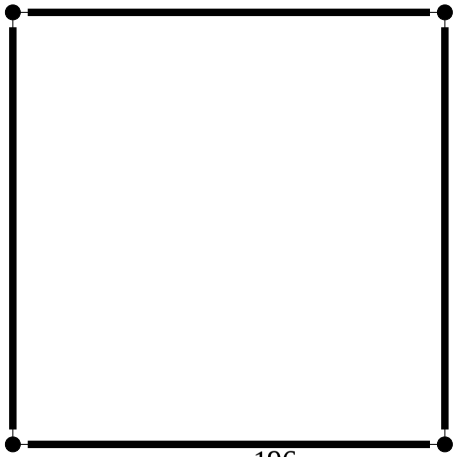
The language of a timed automaton is (omega) regular.

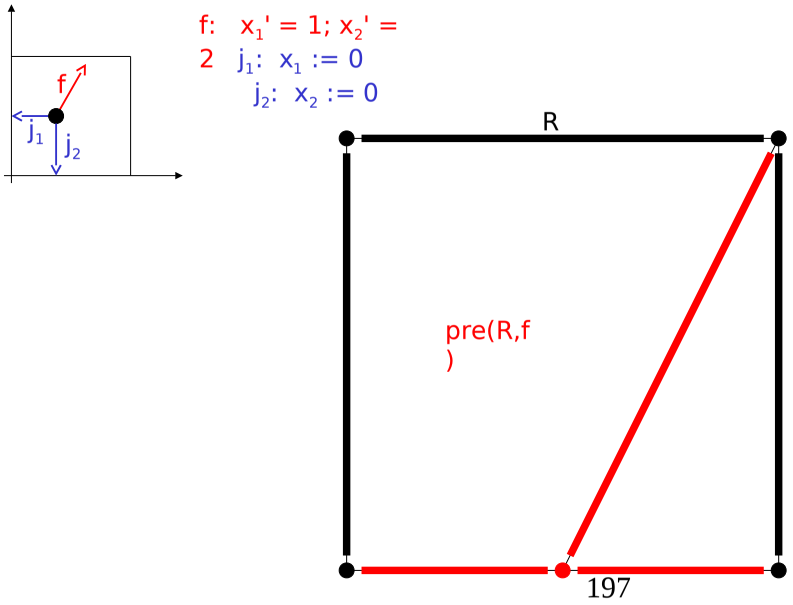
Finite bisimulation.

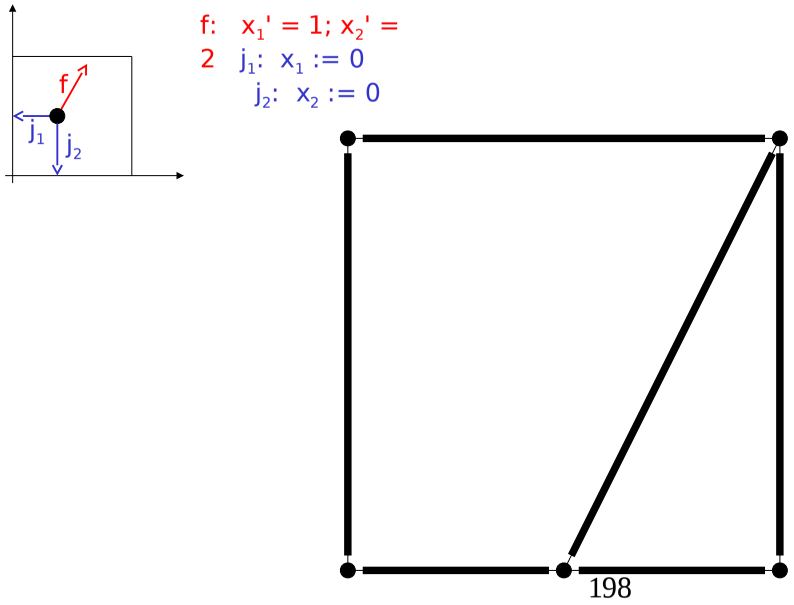


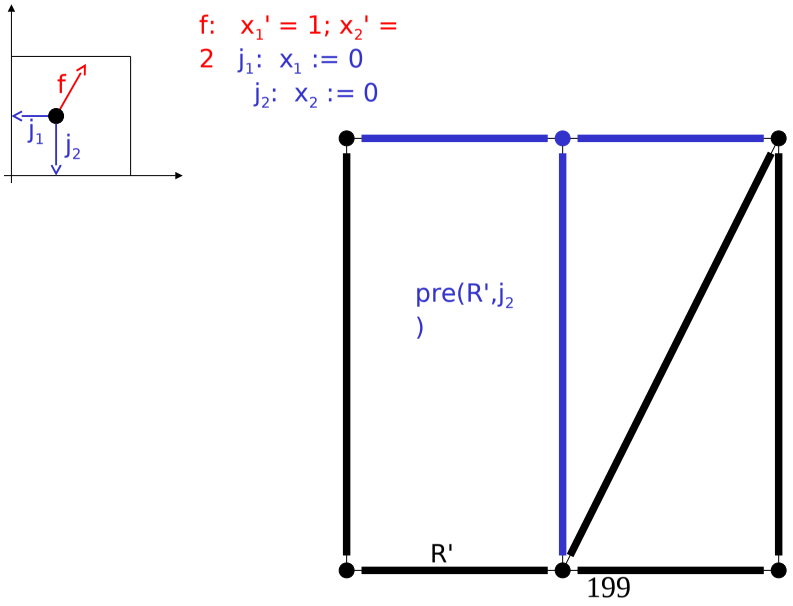


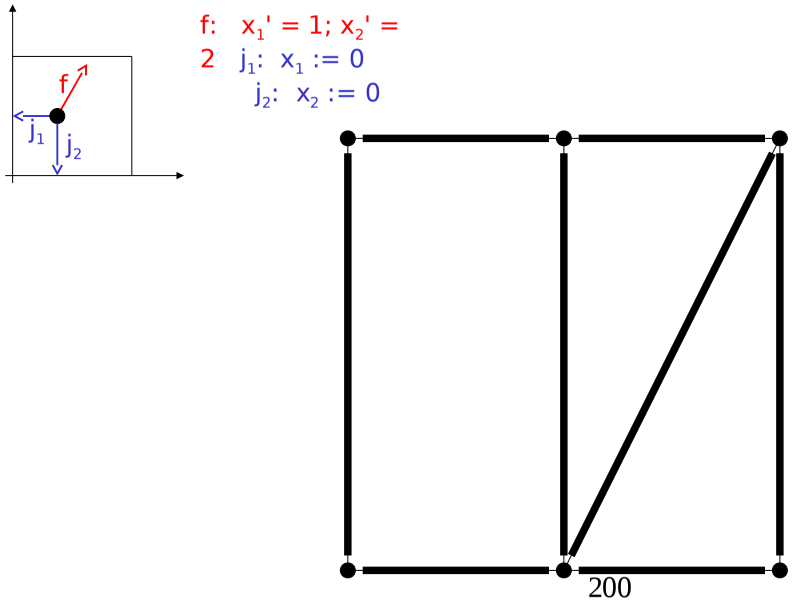
f: $x_1' = 1; x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$

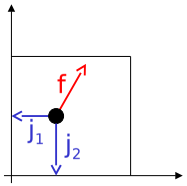



f: x1' = 1; x2' =
2 j1: x1 := 0
j2: x2 := 0
f
j1
j2
R
pre(R,f
)
197

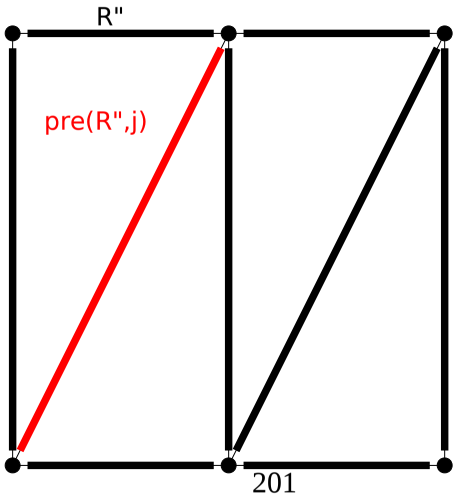

f: x1' = 1; x2' =
2 j1: x1 := 0
j2: x2 := 0
f
j1
j2
198


f: x1' = 1; x2' =
2 j1: x1 := 0
j2: x2 := 0
f
j1
j2
pre(R',j2
)
R'
199

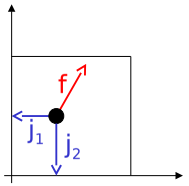

f: x1' = 1; x2' =
2 j1: x1 := 0
j2: x2 := 0
f
j1
j2
200



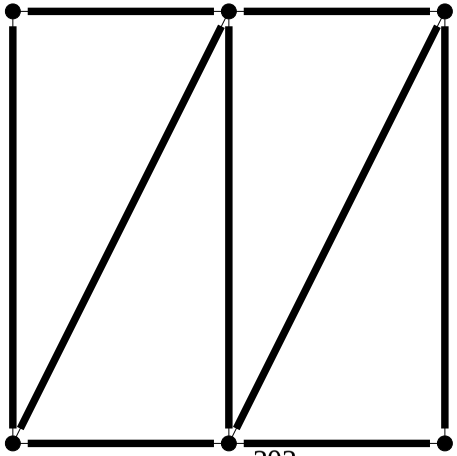
f: $x_1' = 1$; $x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$



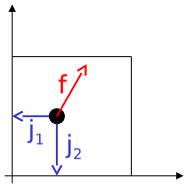
201



f: $x_1' = 1$; $x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$



202



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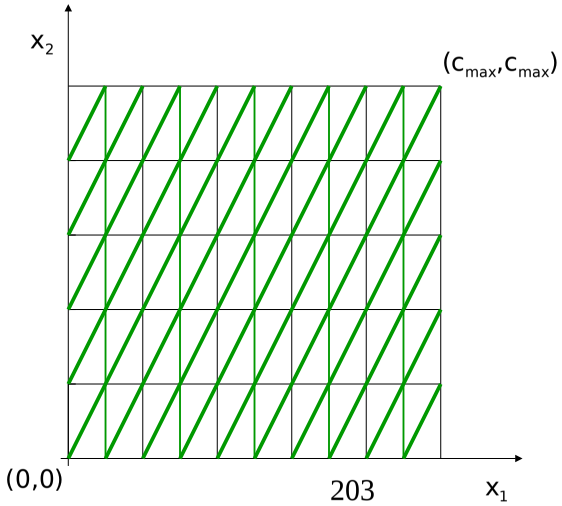
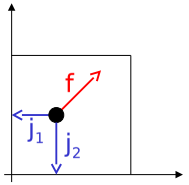


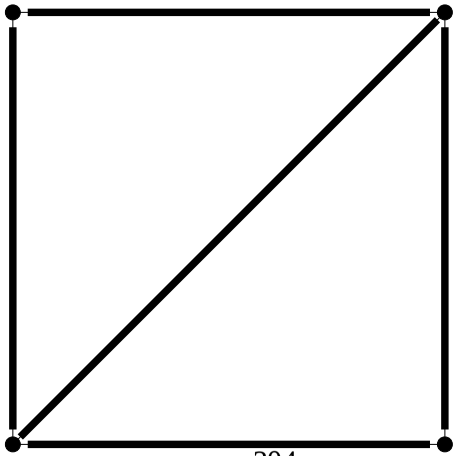
203

Finite bisimulation.

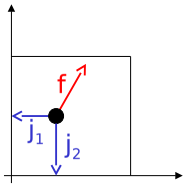


f: $x_1' = 1$; $x_2' =$
1 j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard: $x_1 > x_2$



204



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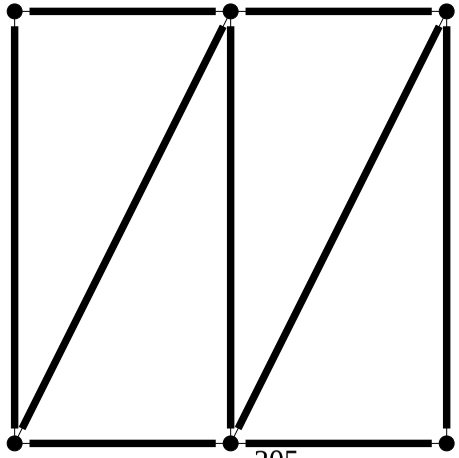
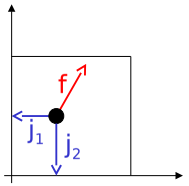
Observation, invariant, or guard:
$x_1 > x_2$



205



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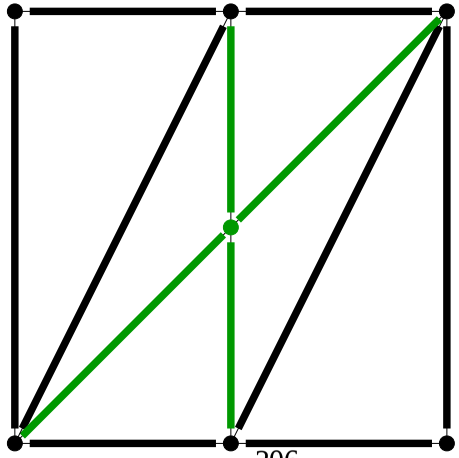
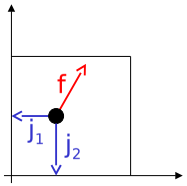
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard:
$x_1 > x_2$



206



f: $x_1' = 1$; $x_2' =$
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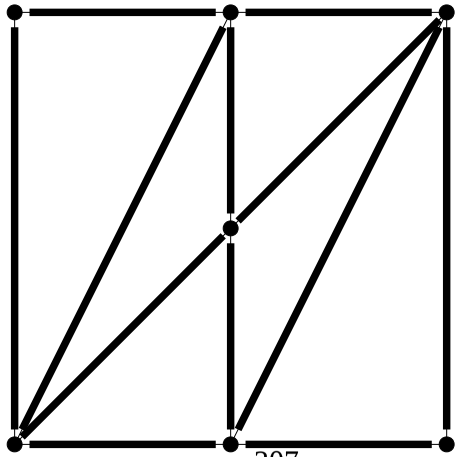
j_2: $x_2 := 0$

Observation, invariant, or guard:
$x_1 > x_2$



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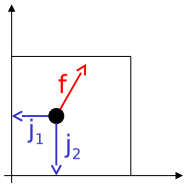
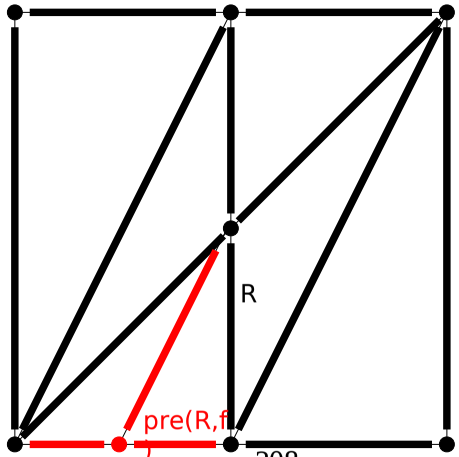


f: $x_1' = 1$; $x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard:
$x_1 > x_2$



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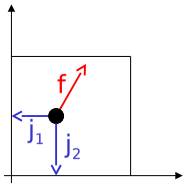

f: $x_1' = 1$; $x_2' =$
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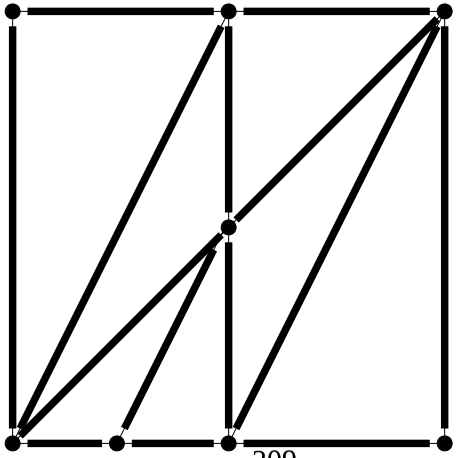
Observation, invariant, or guard:
$x_1 > x_2$



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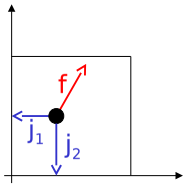

f: $x_1' = 1$; $x_2' =$
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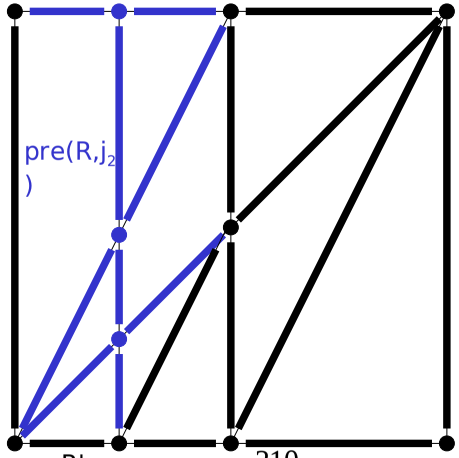
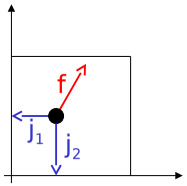
Observation, invariant, or guard:
$x_1 > x_2$

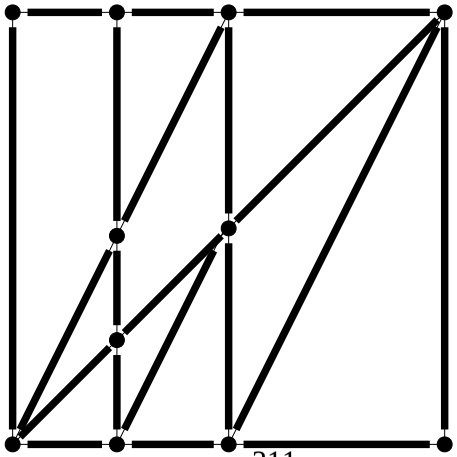


210



f: $x_1' = 1$; $x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard:
$x_1 > x_2$



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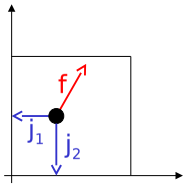
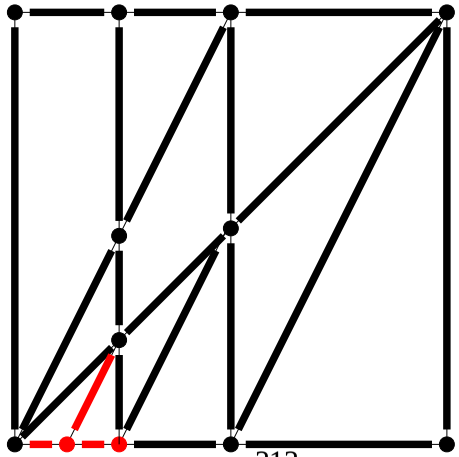


f: $x_1' = 1$; $x_2' =$
2 j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard:
$x_1 > x_2$



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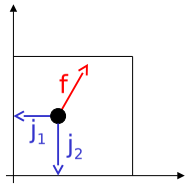
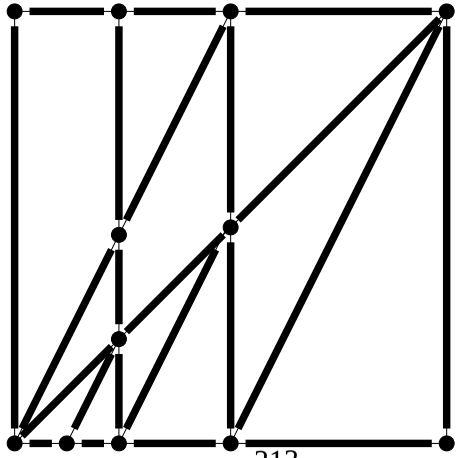
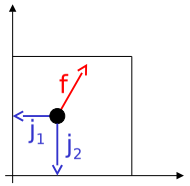


f: $x_1' = 1; x_2' = 2$
j_1: $x_1 := 0$
j_2: $x_2 := 0$

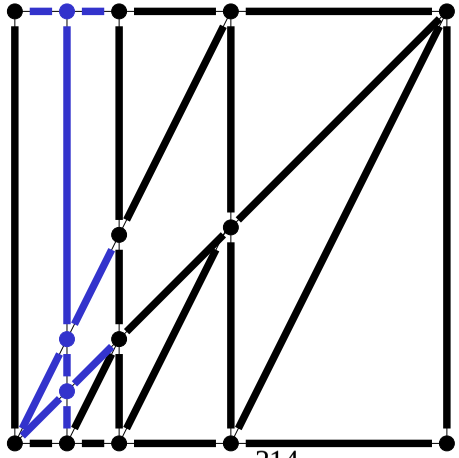
Observation, invariant, or guard: $x_1 > x_2$

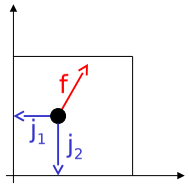




f: $x_1' = 1; x_2' = 2$
j_1: $x_1 := 0$
j_2: $x_2 := 0$

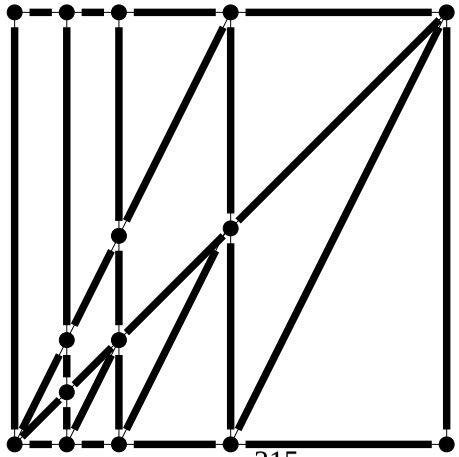
Observation, invariant, or guard: $x_1 > x_2$





f: $x_1' = 1; x_2' = 2$
j_1: $x_1 := 0$
j_2: $x_2 := 0$

Observation, invariant, or guard: $x_1 > x_2$



Infinite bisimulation.

Undecidability of Reachability for Uninitialized Singular Automata

x clock used for storage of counter value

y auxiliary clock

z $z' = 1$ or $z' = 2$

Undecidability of Reachability for Uninitialized Singular Automata

x clock used for storage of counter value

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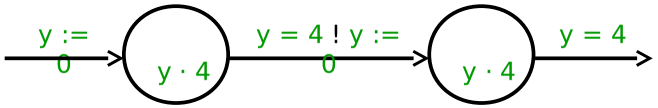
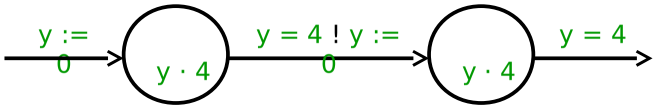
z $z' = 1$ or $z' = 2$

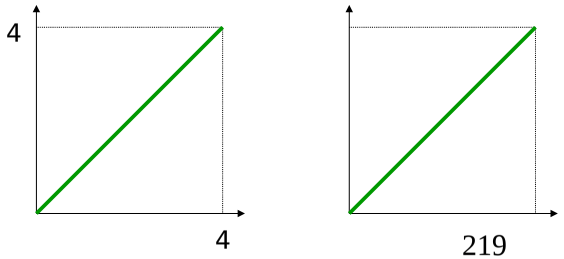
Encoding of counter value a:
every 4 time units, $x = 1 / 2^a$

Doubling of Clock Value

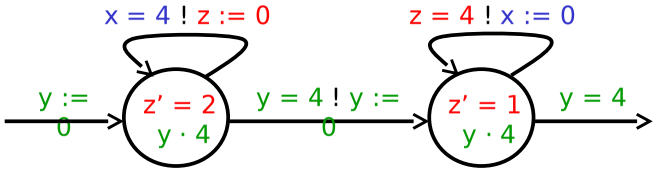


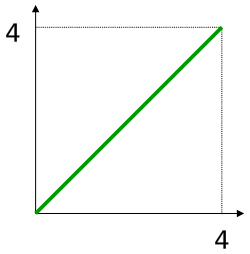
Doubling of Clock Value

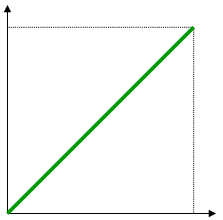




Doubling of Clock Value







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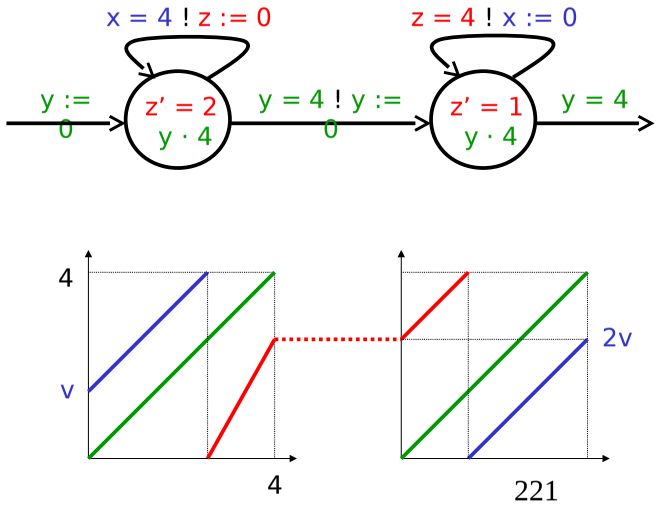
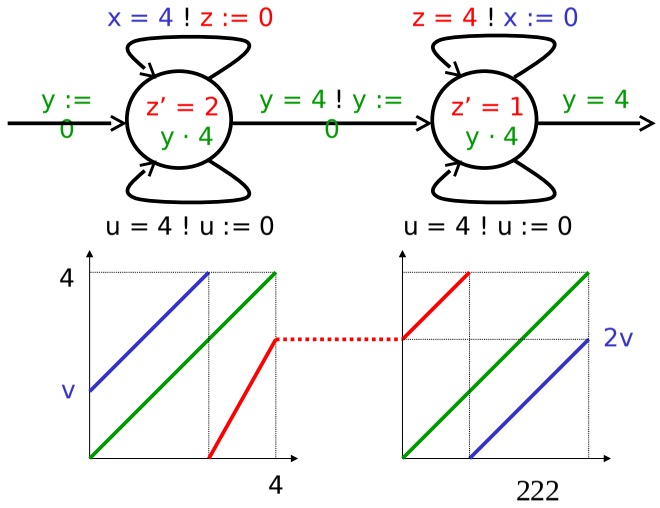
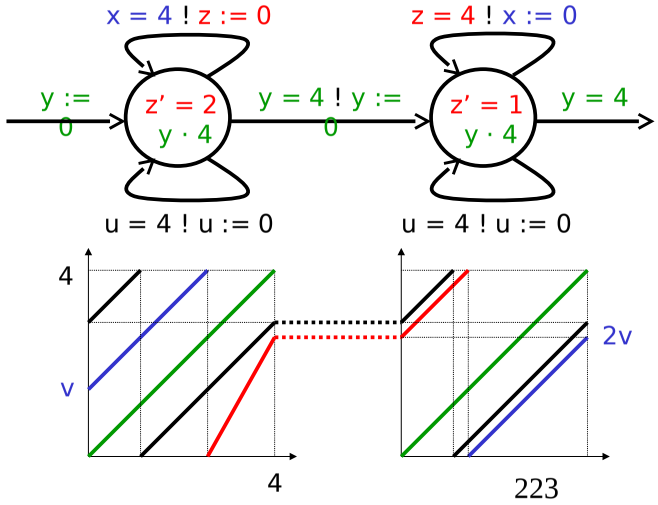


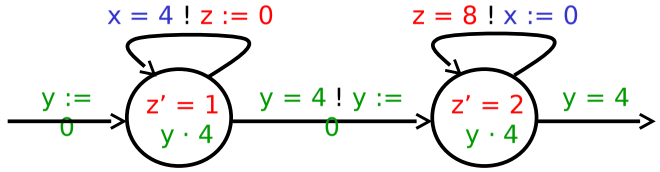
Doubling of Clock Value



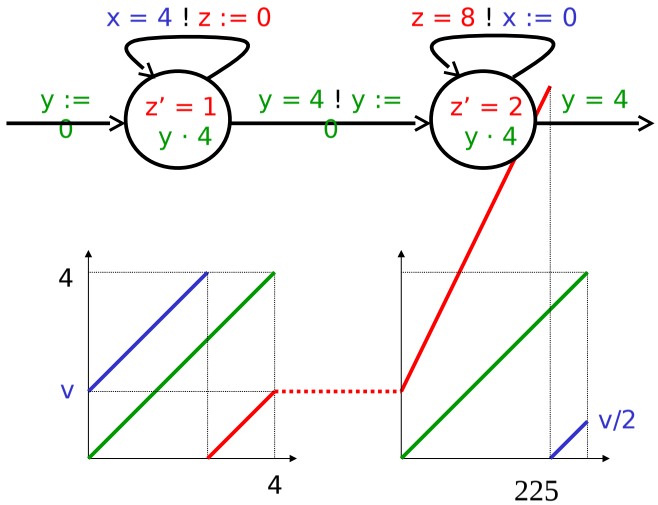
Doubling of Clock Value



Halving of Clock Value



Halving of Clock Value



Example: Rectangular Hybrid Automata

$Q = B^m \times R^n$

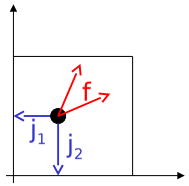
Invariants and guards:
integral bounds, e.g. $x_1 < 7 \wedge 1 \leq x_2 \leq 2$

Flows: bounded slopes, e.g. $x'_1 \in [1,2]$; $x'_2 = 1$

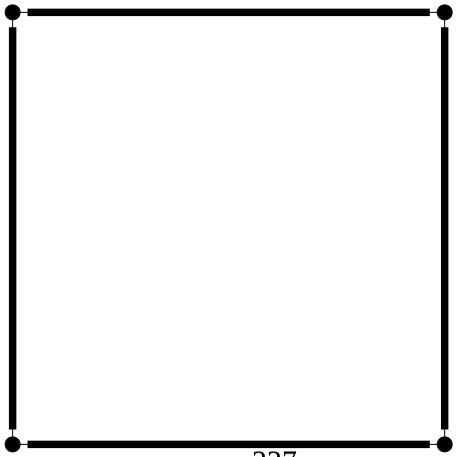
Jumps: integral assignments, e.g. $x_1 := 0$; $x_2 := 5$

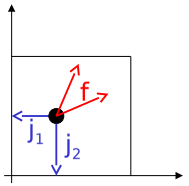
$A = \{ x_i = c, c < x_i < c+1 \mid 1 \leq i \leq n, c \in N, c < c_{max} \}$

Initialized: assignment when slope bounds change.



f: $x_1' \in [1,2]$;
$x_2' \in [1,2]$ j_1: $x_1 := 0$
j_2: $x_2 :=$
0





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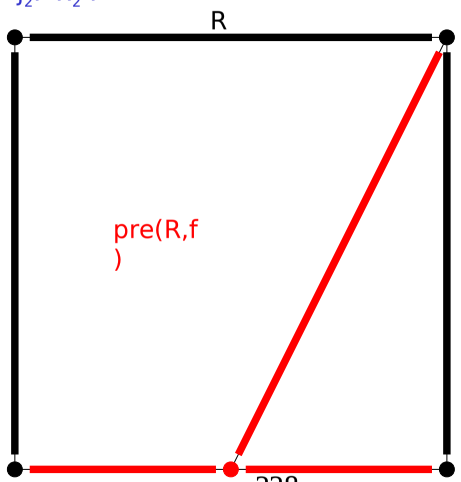

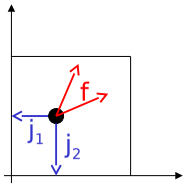

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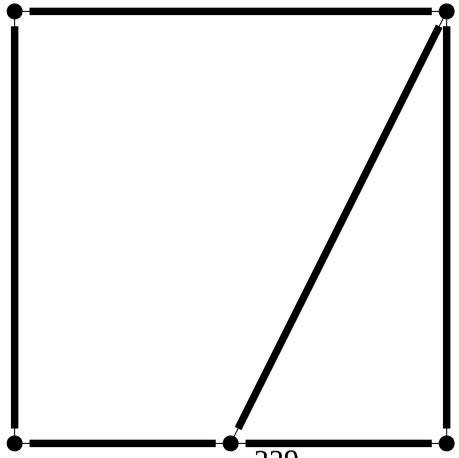
0



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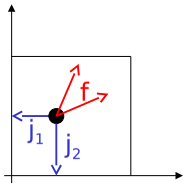

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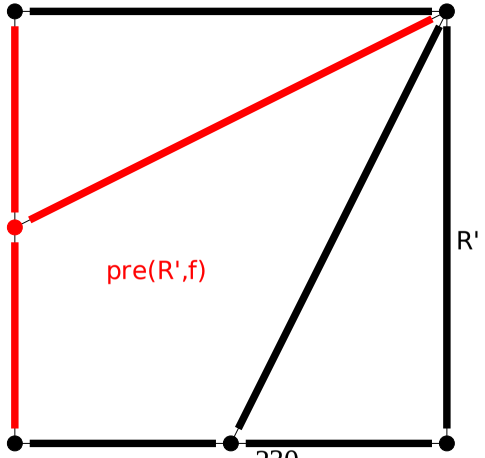

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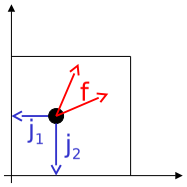

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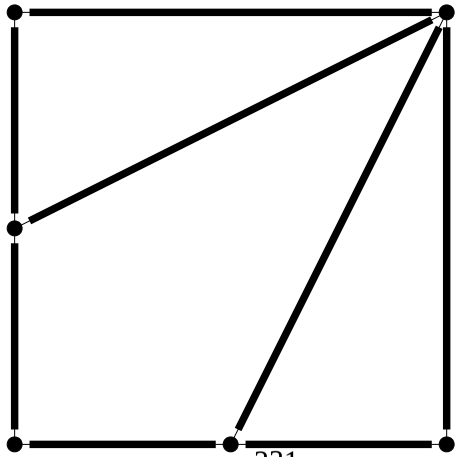
j_2: $x_2 :=$
0



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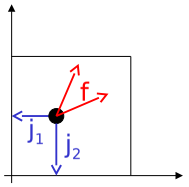

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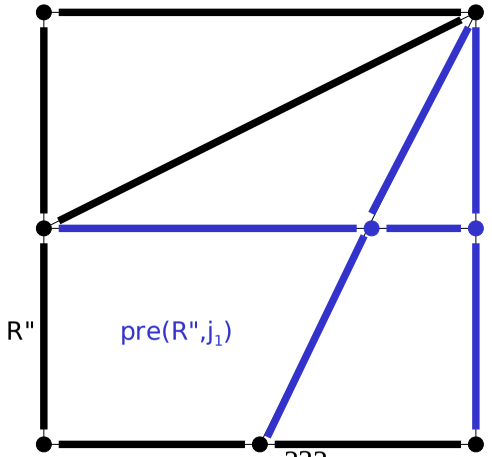

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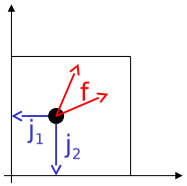

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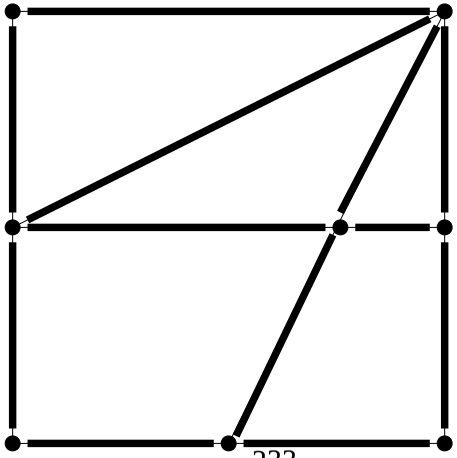

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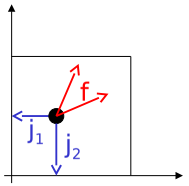

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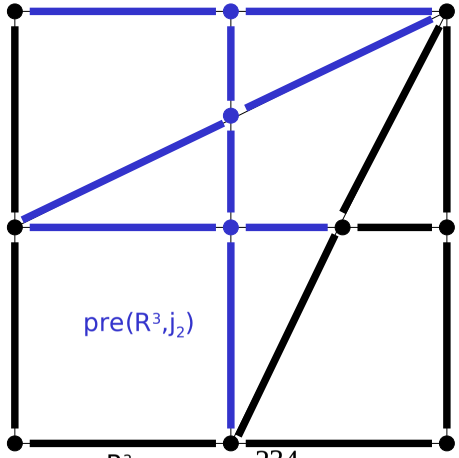
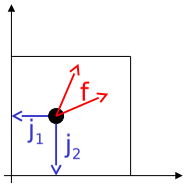


R^3 234



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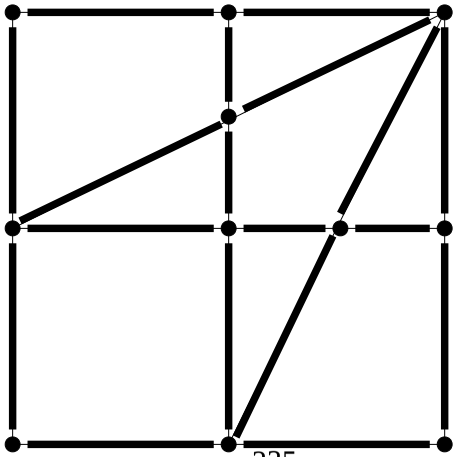

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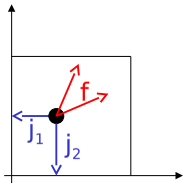

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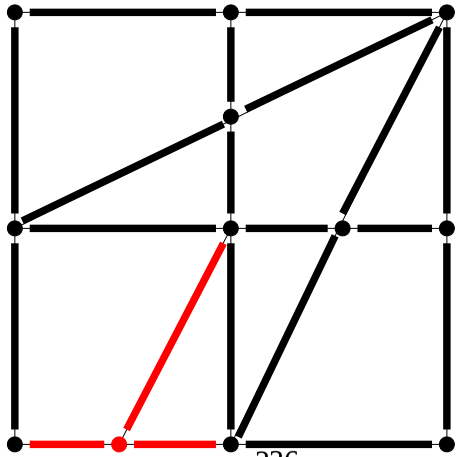

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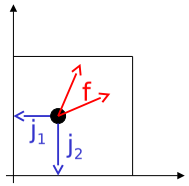

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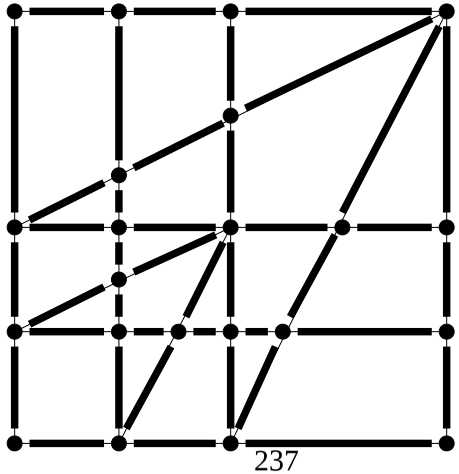
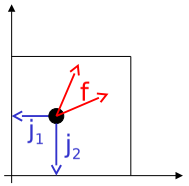
Infinite bisimulation.





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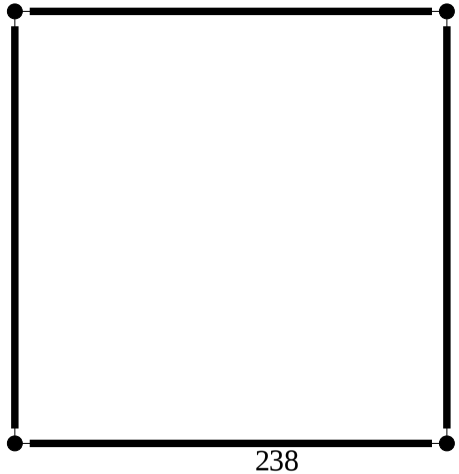

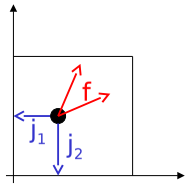

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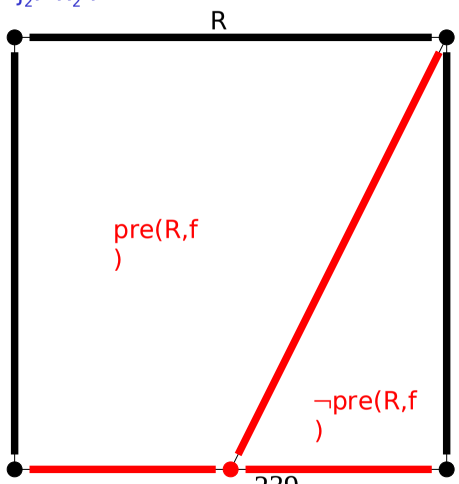

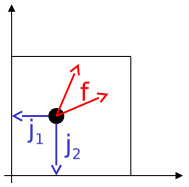

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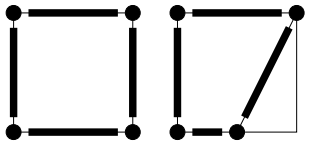

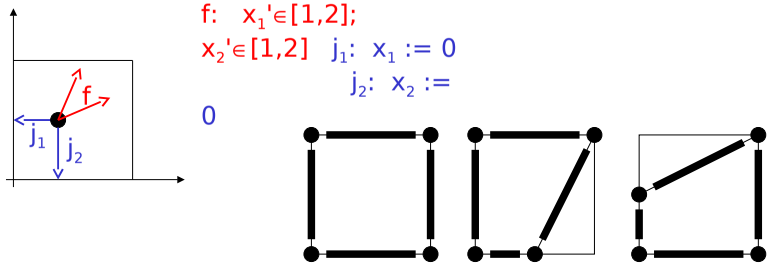

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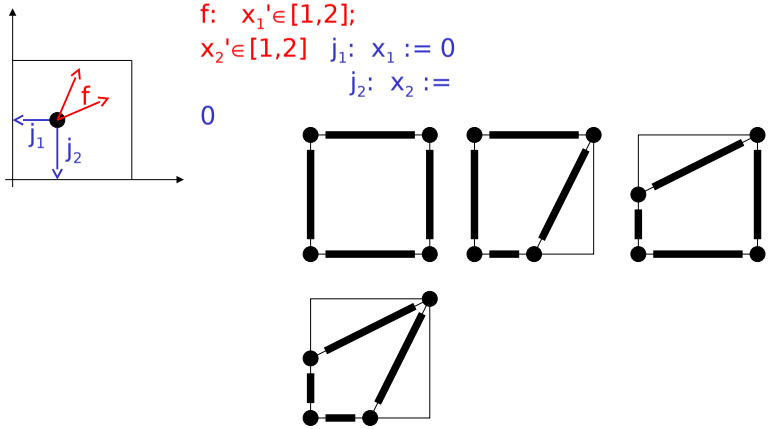

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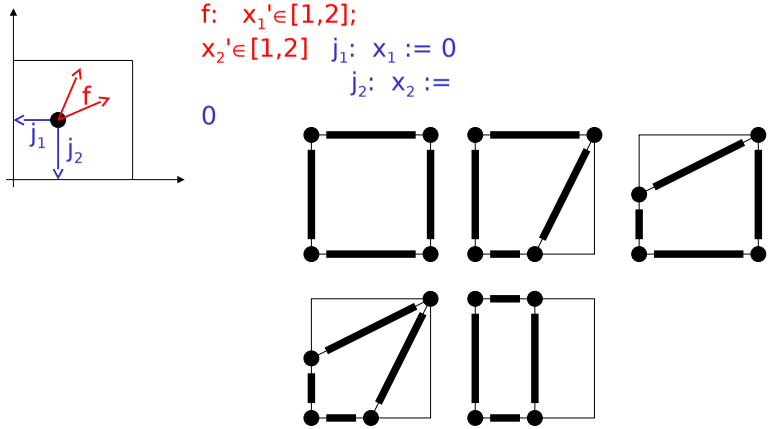

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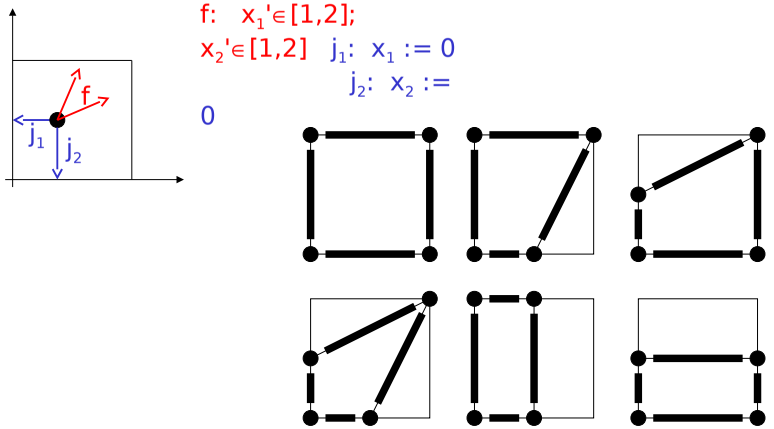

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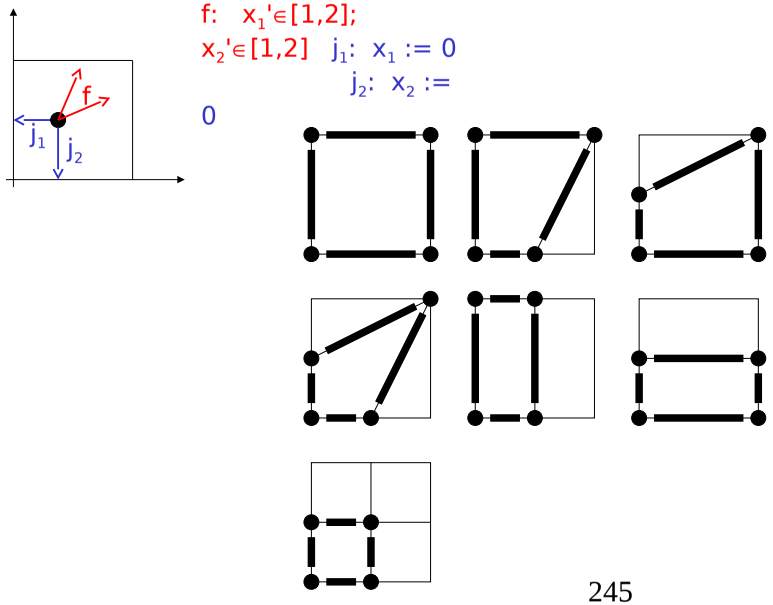
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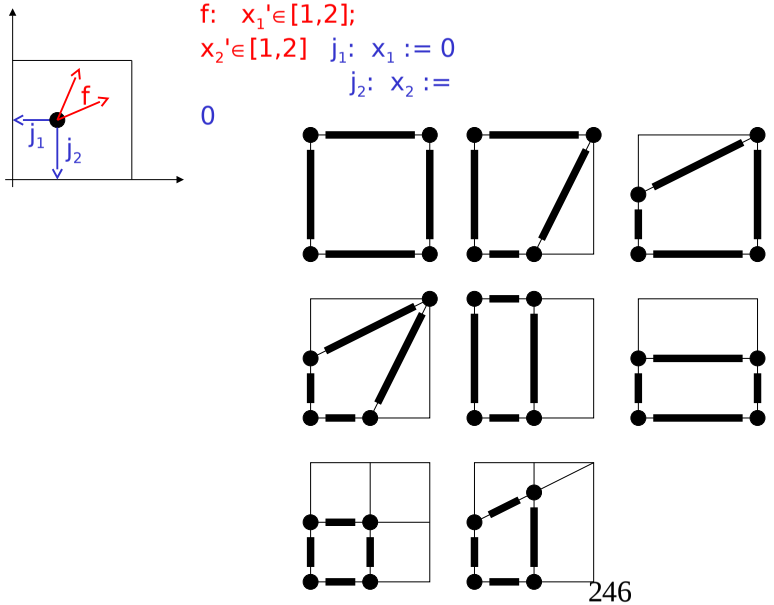
j2: x2 :=
0
f
j1
j2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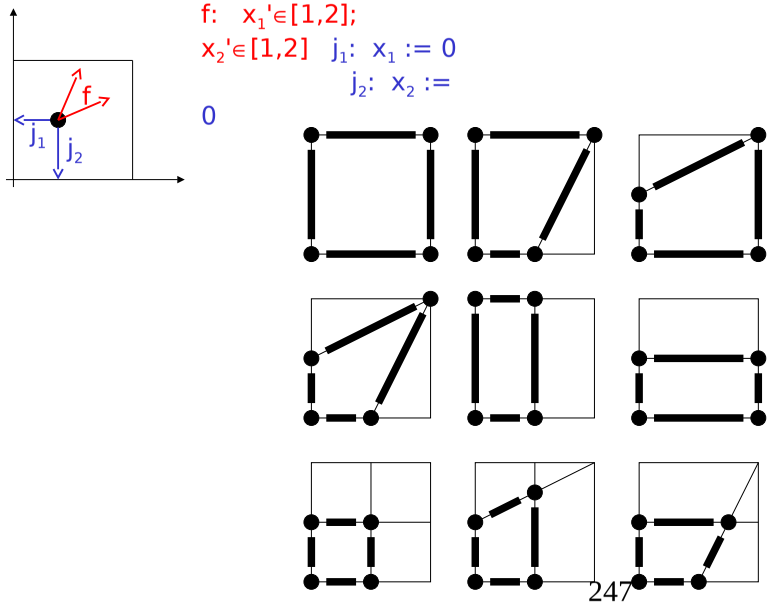
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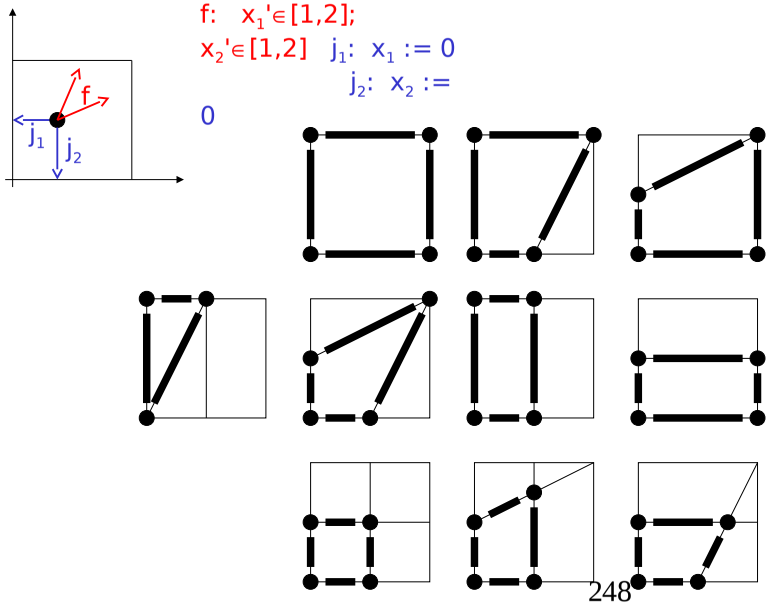
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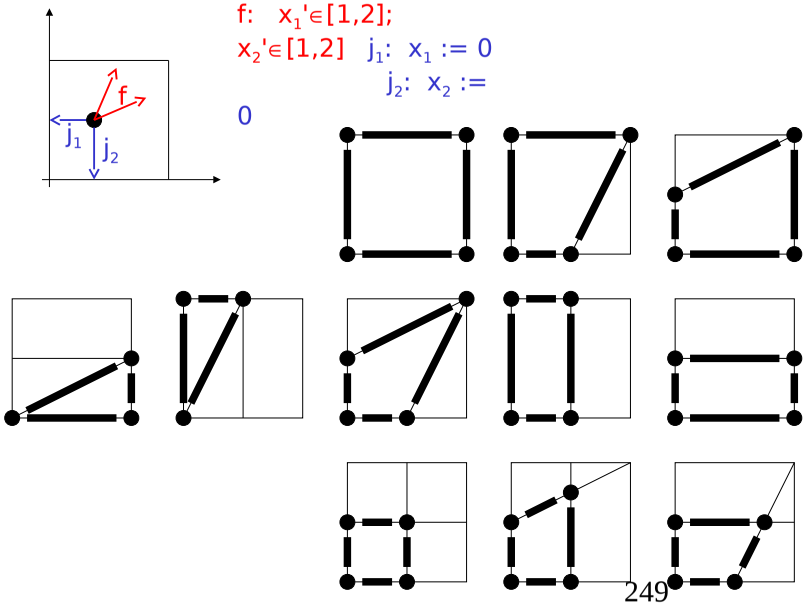

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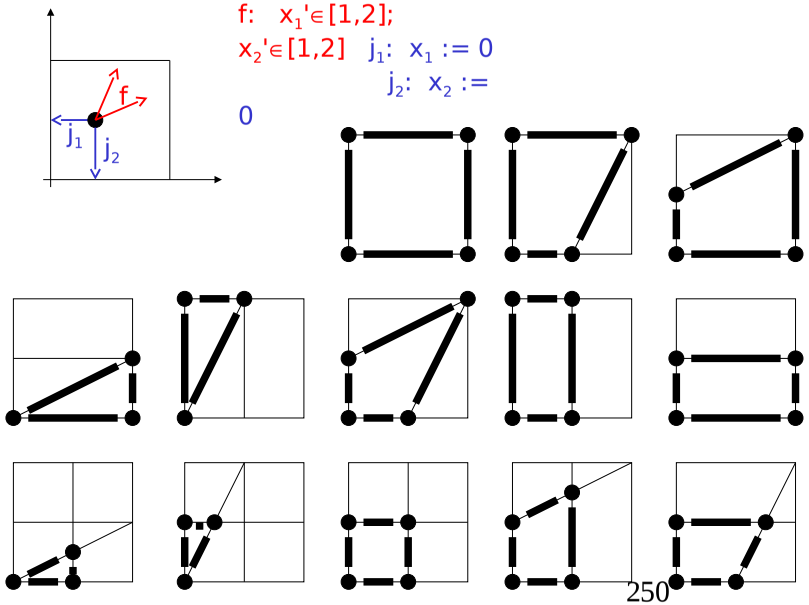

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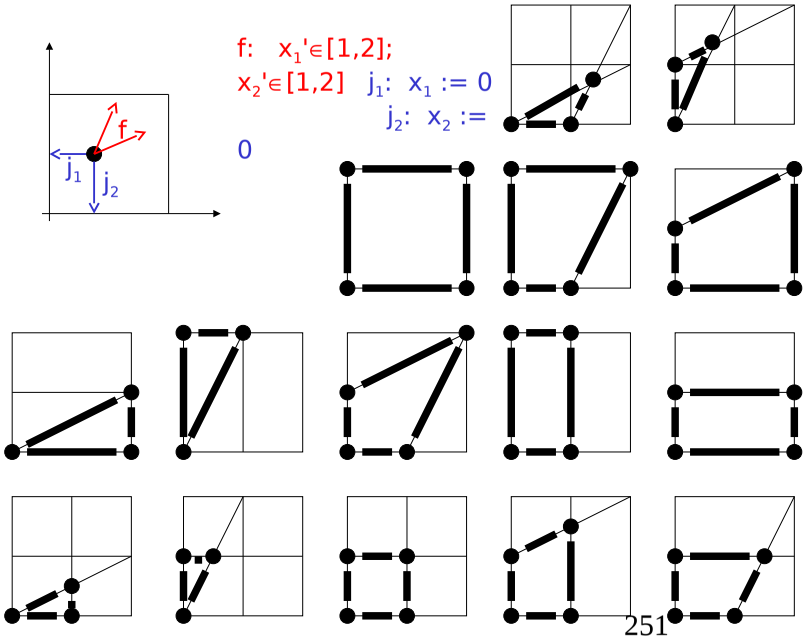

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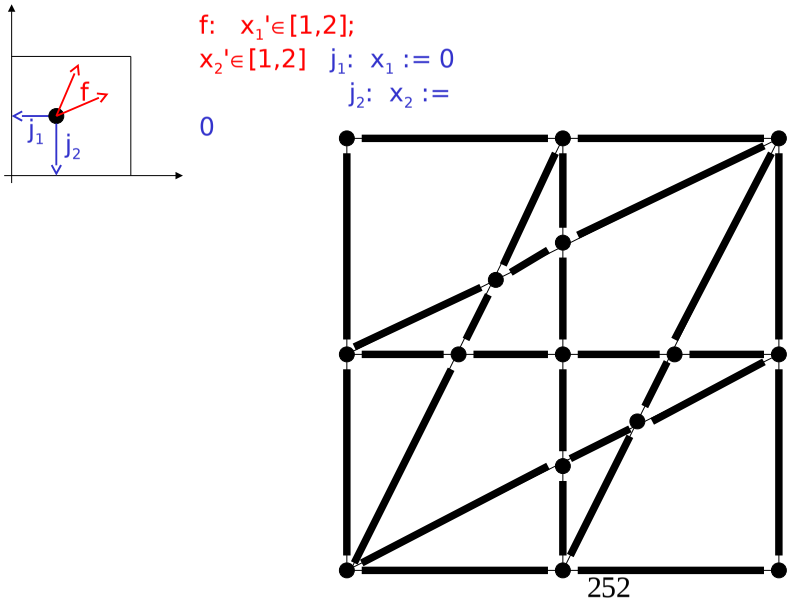

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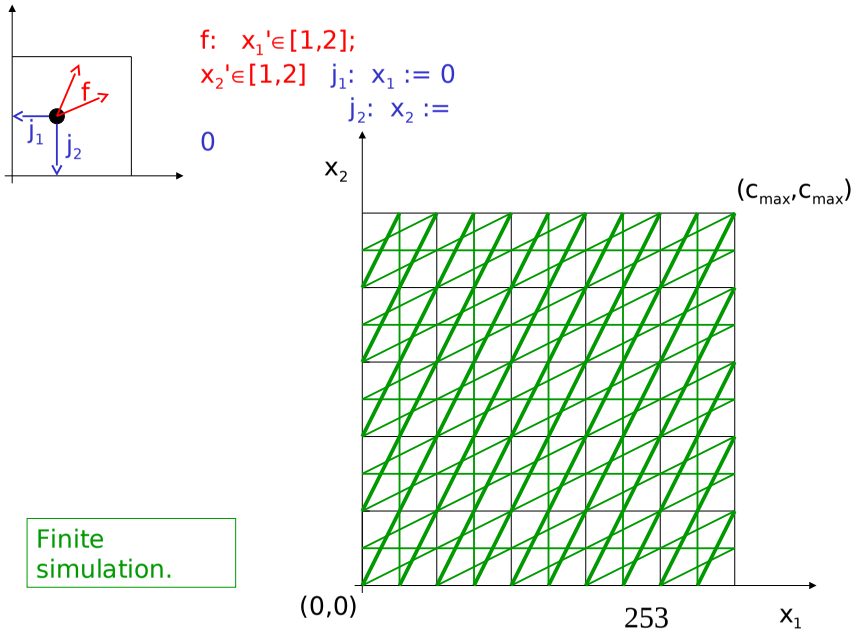


Finite simulation.

Summary

Timed and initialized singular automata:

STS5 ⇒ CTL model checking
[Alur, Dill; Alur, Courcoubetis, H, Ho]

2D initialized rectangular automata:

STS4 ⇒ ∀CTL model checking
[H, Kopke]

Initialized rectangular automata:

STS3 ⇒ LTL model checking
[H, Kopke, Puri, Varaiya]

Networks of timed automata:

STS1 ⇒ reachability analysis
[Abdullah, Jonsson]

Two Messages for Infinite-State Model Checking

1. Separate local (region algebra) from global (symbolic semi-algorithm) concerns

2. Appeal to finite abstractions in the termination argument, not in the algorithm

Suppose a hybrid system consists of several components (e.g., controller and plant).

V1-5: Can the components collaborate to achieve an objective?

C1-5: Can a subset of the components (e.g., the controller) achieve the objective no matter how the other components (the plant) behave?

Need model that preserves components: "players" in a concurrent game.

The Thermostat Revisited

Player 1 (plant):

States

$x \in R$ temperature

Inputs

$h \in \{ on, off \}$ heat

Flows

f_1 $[][]h = on \rightarrow x' = K\cdot(H-x)$

f_2 $[] h = off \rightarrow x' = -K\cdot x$

Jumps

The Thermostat Revisited

Payer 2 (controller);

States

$h \in \{ on, off \}$ heat

$t \in R$ timer

Flows

f $[] t \le U \rightarrow t' = 1$

Jumps

j_1 $[] h = on \wedge t \ge L \rightarrow h := off; t := 0$

j_2 $[] h = off \wedge t \ge L \rightarrow h := on; t := 0$

Concurrent Game

Q states

Σ_1, Σ_2 moves of both players

post: $Q \times \Sigma_1 \times \Sigma_2 \rightarrow Q$ transitions

Concurrent Game

Q states

Σ_1, Σ_2 moves of both players

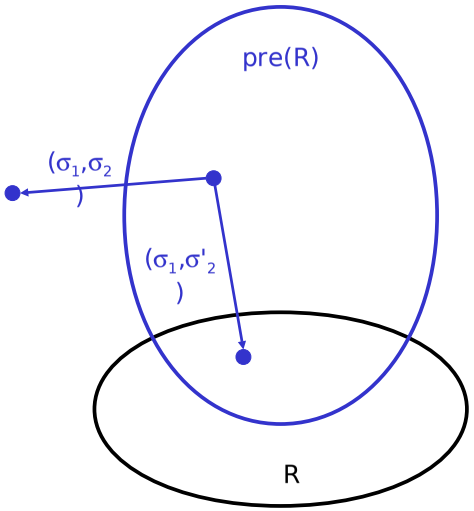
post: $Q \times \Sigma_1 \times \Sigma_2 \rightarrow Q$ transitions

$cpre_1: 2^Q \times \Sigma_1 \rightarrow 2^Q$

$q \in cpre_1(R,\sigma_1)$ iff for all $\sigma_2 \in \Sigma_2$,

$post(q,\sigma_1,\sigma_2) \in R$

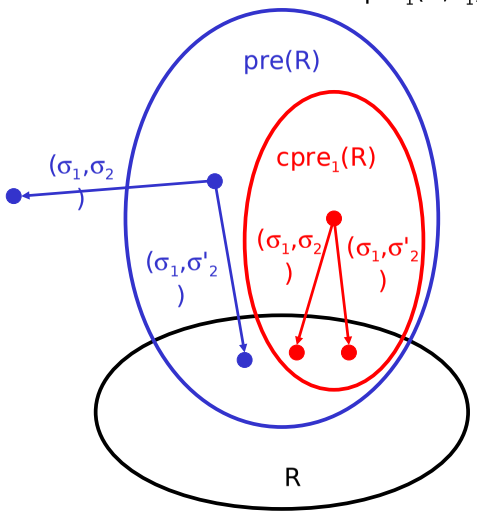
Σ_1 = $\{\sigma_1,\sigma'_1\}$ Σ_2 = $\{\sigma_2,\sigma'_2\}$



261

Σ_1 = $\{\sigma_1,\sigma'_1\}$ Σ_2 = $\{\sigma_2,\sigma'_2\}$

$cpre_1(R) = \cup_{\sigma 12 \Sigma 1} cpre_1(R,\sigma_1)$



262

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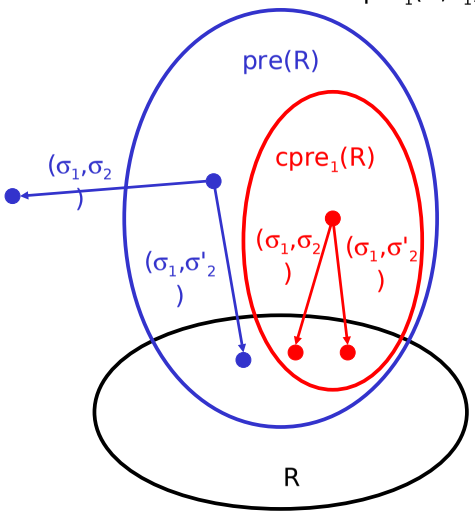

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98

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Concurrent Game

Q states

Σ_1, Σ_2 moves of both players

post: $Q \times \Sigma_1 \times \Sigma_2 \rightarrow$ Q transitions

$cpre_1$: $2^Q \times \Sigma_1 \rightarrow 2^Q$

$q \in cpre_1(R,\sigma_1)$ iff for all $\sigma_2 \in \Sigma_2$, $post(q,\sigma_1,\sigma_2) \in R$

$cpre_2$: $2^Q \times \Sigma_2 \rightarrow 2^Q$

$q \in cpre_2(R,\sigma_2)$ iff for all $\sigma_1 \in \Sigma_1$,

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Symbolic Concurrent Game

Q states

Σ_1, Σ_2 moves of both players

$cpre_1, cpre_2$ controllable pre operators

A observations

$\Re = \{ R_1, R_2, \ldots \}$ regions $R_i \subseteq Q$

Region algebra:

1. $A \subseteq \Re$
2. $cpre_1: \Re \times \Sigma_1 \to \Re$ computable $cpre_2: \Re \times \Sigma_2 \to \Re$ computable
3. $\AA : \Re^2 \to \Re$
   $\setminus : \Re^2 \to \Re$ computable
   $\subseteq : \Re^2 \to \{ t, f \}$

Symbolic Semi-Algorithm

Starting from the observations in A, compute new regions in $\Re$ by applying the operations $cpre_1$, $cpre_2$, $\AA$, \ , and $\subseteq$.

V1: Verification for Reachability

$a \wedge \exists\Box\, b$

Given a,b∈A, is there a trajectory from a to b?

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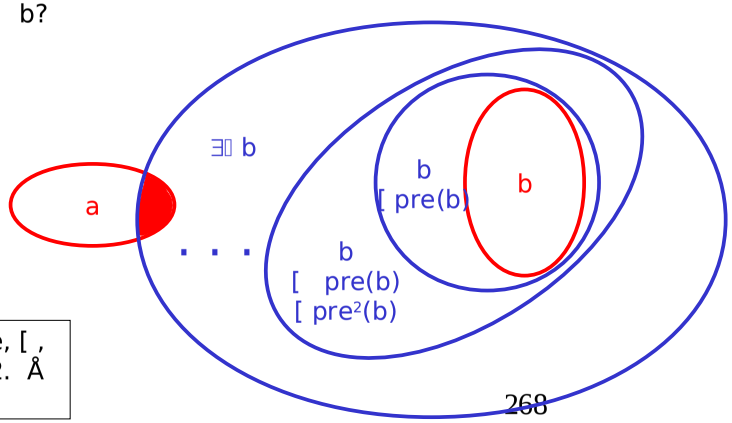

1. pre, [, ⊆ 2. $\AA$ a

C1: Control for Reachability

$a \wedge \langle\langle 1 \rangle\rangle \Box b$

Given a,b∈A, does player 1 have a strategy to force the game from a to b?

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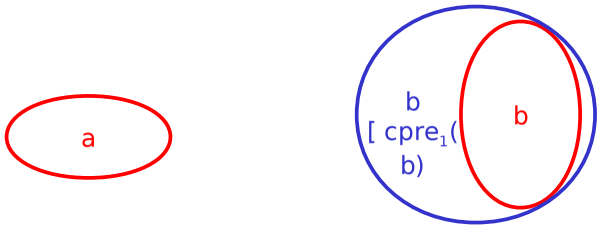

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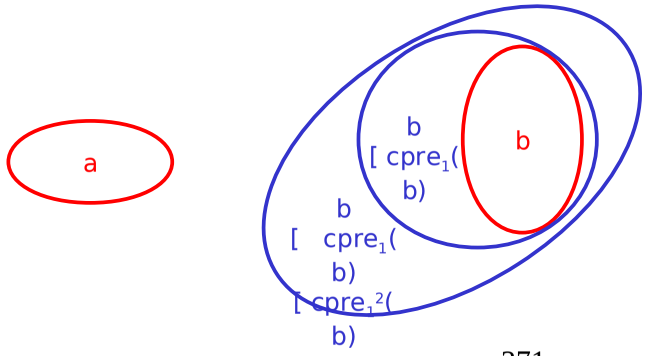

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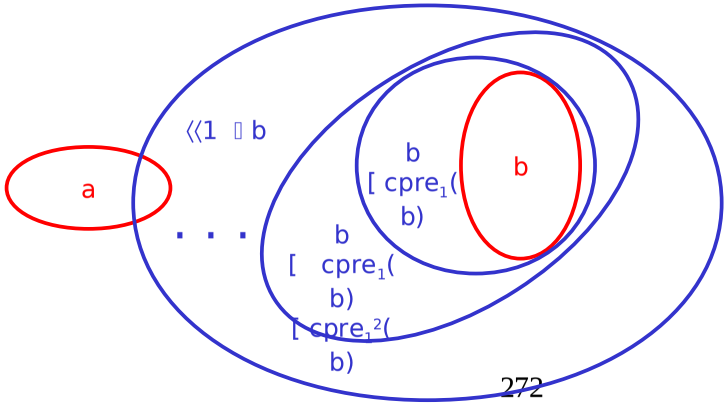

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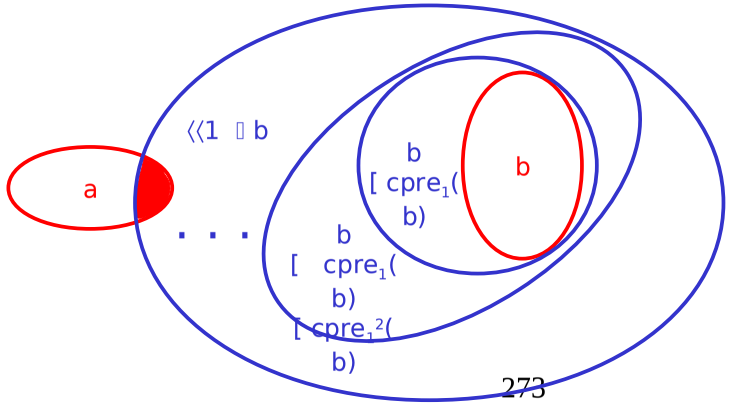

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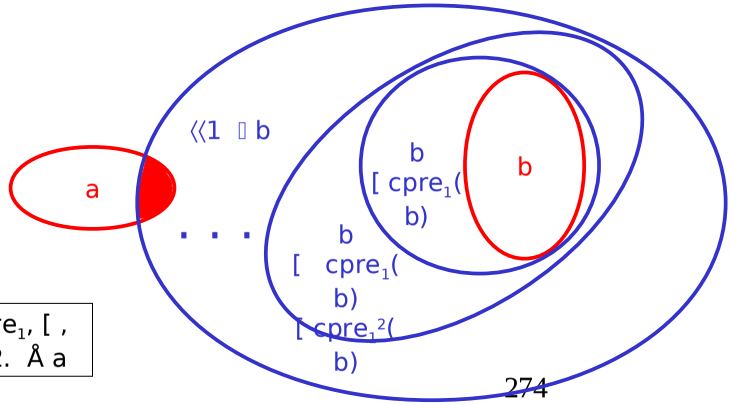

1. cpre1, [, ⊆ 2. Å a

V3: Buechi Verification

a ∧ ∃□⟩ b

Given a,b∈A, is there an infinite trajectory from a that visits b infinitely often?

V3: Buechi Verification

a ∧ ∃□⟩ b

Given a,b∈A, is there an infinite trajectory from a that visits b infinitely often?

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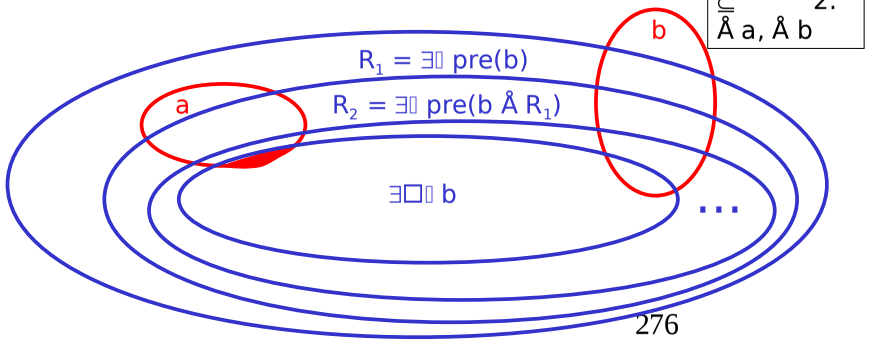

C3: Buechi Control

$a \wedge \langle\langle 1 \rangle\rangle \Box\Diamond b$

Given a,b∈A, does player 1 have a strategy from a to visit b infinitely often?

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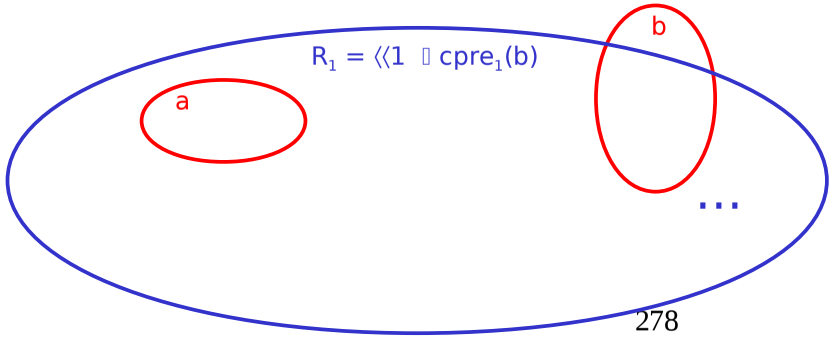

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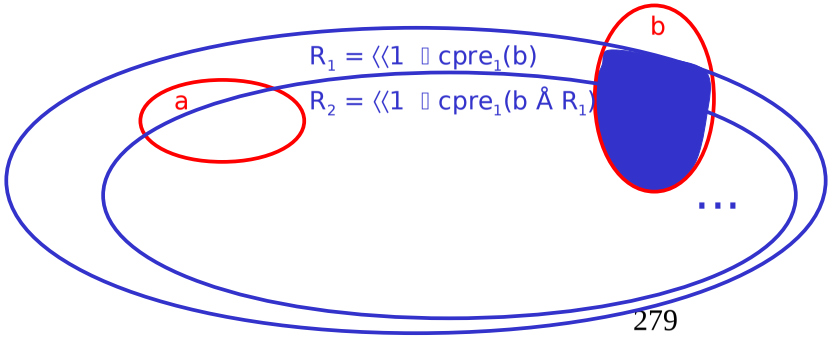

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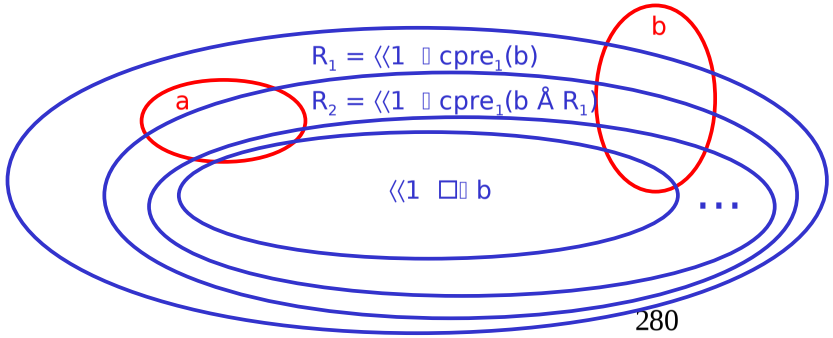

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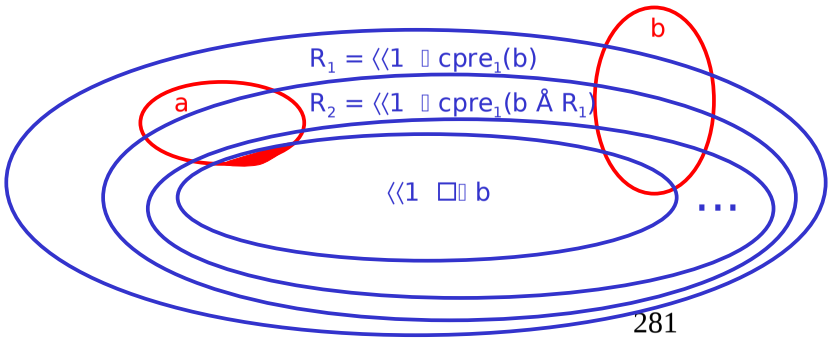

C3: Buechi Control

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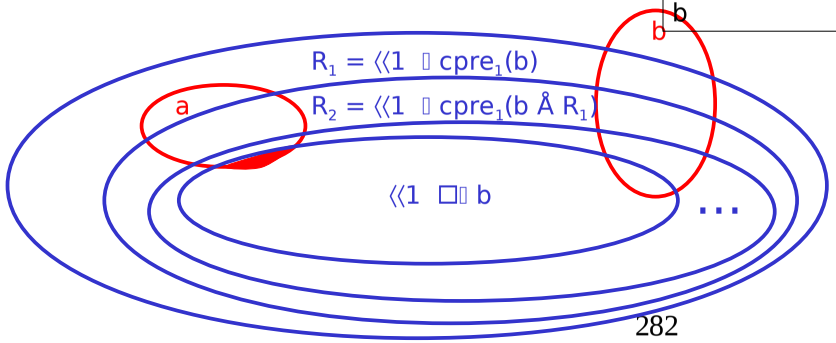
1. cpre_1, [, ⊆
2. Å a, Å b



q_1 is simulated by q_2

iff

there is a simulation relation S such that

1. $S(q_1,q_2)$
2. if S(p,q) then
   a. (8 a2A) (p2 a iff q2 a)
   b. (8 p') (if p2 pre(p') then (9 q') (q2 pre(q') Æ S(p',q')))

q_1 is alternating simulated by q_2 [Alur, H, Kupferman, Vardi]

iff

there is an alternating simulation relation S such that

1. $S(q_1,q_2)$
2. if S(p,q) then
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   b. (8 p') (if p2 cpre_1(p') then (9 q') (q2 cpre_1(q') Æ S(p',q')))

Five Classes of Symbolic Concurrent Games

SCG1: $cpre_1$ iteration terminates ⇒
⟨⟨1 ▯ decidable

SCG2: $cpre_1$ closure terminates ⇒
conjunction-free alternating μ-calculus decidable

SCG3: ($cpre_1$, Å a) terminates ⇔ Finite alternating 1-trace equiv ⇒ guarded alternating μ-calculus (LTL, omega games) decidable

SCG4: ($cpre_1$, Å) terminates ⇔ Finite alternating 1-similarity ⇒ existential alternating μ-calculus (⟨⟨1 ATL) decidable

SCG5: ($cpre_1$, Å , \ ) terminates ⇔ Finite alternating 1-bisimilarity ⇒ alternating μ-calculus (ATL) decidable

Summary

Timed and initialized singular automata:

SCG5 ⇒ ATL control
[de Alfaro, H, Majumdar]

2D initialized rectangular automata:

SCG4 ⇒ ⟨⟨1 ATL control
[de Alfaro, H, Majumdar]

Initialized rectangular automata:

SCG3 ⇒ LTL control
[H, Horowitz, Majumdar]

Networks of timed automata:

SCG1 ⇒ reachability control

Verification vs. Control: Can we use the "same" algorithms?

Vϕ

Cϕ

Suppose we have an LTL formula ϕ and a symbolic semi-algorithm A(pre) that computes 9ϕ.

Question: does A($cpre_1$) compute ⟨⟨1 ϕ, that is, does it solve the game with player-1 objective ϕ?

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Not necessarily!

From Verification to Control

Thm 1: If A(pre) computes 9ϕ and A(pre) computes 8ϕ, then A($cpre_1$) computes $\langle\langle 1 \rangle\rangle \phi$.

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Thm 2: For every LTL formula ϕ, we can construct a symbolic semi-algorithm (i.e., guarded μ-calculus formula) A_ϕ that satisfies the premise of Thm 1.

[de Alfaro, H, Majumdar: LICS 2001]

Two Messages for Infinite-State Model Checking and Control

1. Separate local (region algebra) from global (symbolic semi-algorithm) concerns

2. Appeal to finite abstractions in the termination argument, not in the algorithm